

Data versus theory update

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Data

CERES EBAF Ed4.1, 240 months (Dec 2000 – Nov 2020)

Theory

Schwarzschild (1906, Eq. 11)

Theory

On the Equilibrium of
the Sun's Atmosphere

Schwarzschild (1906)

Radiative equilibrium

Two-stream

A upward beam

B downward beam

E emission of the layer

Ueber das Gleichgewicht der Sonnenatmosphäre

Von

K. Schwarzschild.

Vorgelegt in der Sitzung vom 13. Januar 1906.

c) *Radiative Equilibrium.* If we assume that the outer regions of the sun

Assume that each layer dh of the solar atmosphere absorbs a fraction adh of the transmitted radiation. If E is the emission of a black body at the temperature of this layer dh , and assuming that Kirchhoff's law applies, it follows that this layer radiates the energy $aEdh$ in every direction.

Consider now, at some point in the solar atmosphere, the radiative energy A which is transmitted outward, and the radiative energy B , which (due to the radiation of outer layers) is transmitted inward.

Treat first the inward energy B . When traveling inward through an infinitesimally thin layer dh , the fraction $aBdh$ of B will be lost; on the other hand, the contribution $aEdh$ due to the lateral radiation of the layer itself will be added to B . All in all,

$$\frac{dB}{dh} = a(E - B). \quad (7)$$

K. Schwarzschild

In the case of the outward energy A , we proceed analogously and obtain

$$\frac{dA}{dh} = -a(E - A). \quad (8)$$

Given the absorption coefficient a as a function of depth h , define the “average optical depth”* of the atmosphere lying above the depth h by

$$\bar{\tau} = \int^h a dh. \quad (9)$$

The differential equations then become

$$\frac{dB}{d\bar{\tau}} = E - B, \quad \frac{dA}{d\bar{\tau}} = A - E. \quad (10)$$

We want to find the temperature distribution under steady-state conditions. These require that each layer receives as much energy as it radiates, i.e., that

$$aA + aB = 2aE, \quad A + B = 2E.$$

Introducing the parameter ζ such that

$$A = E + \zeta, \quad B = E - \zeta,$$

we obtain the differential equations in the form

$$\frac{d\zeta}{d\bar{\tau}} = 0, \quad \frac{dE}{d\bar{\tau}} = \zeta,$$

and after integration we have

$$\begin{aligned}\zeta &= \text{const.} \\ E &= E_0 + \zeta\bar{\tau} \\ A &= E_0 + \zeta(1 + \bar{\tau}) \\ B &= E_0 + \zeta(\bar{\tau} - 1).\end{aligned}$$

The constants of integration E_0 and ζ are fixed by the requirements that there can be no inward radiation at the outer boundary of the atmosphere ($\bar{\tau} = 0$), and that the outward energy there must have the observed value A . Thus at $\bar{\tau} = 0$ we must have

$$B = 0, \quad A = A_0.$$

This leads to the final result

$$E = \frac{A_0}{2} (1 + \bar{\tau}), \quad A = \frac{A_0}{2} (2 + \bar{\tau}), \quad B = \frac{A_0}{2} \bar{\tau}. \quad (\text{II})$$

$$E = \frac{A_0}{2} (1 + \bar{\tau}), \quad A = \frac{A_0}{2} (2 + \bar{\tau}), \quad B = \frac{A_0}{2} \bar{\tau}. \quad (11)$$

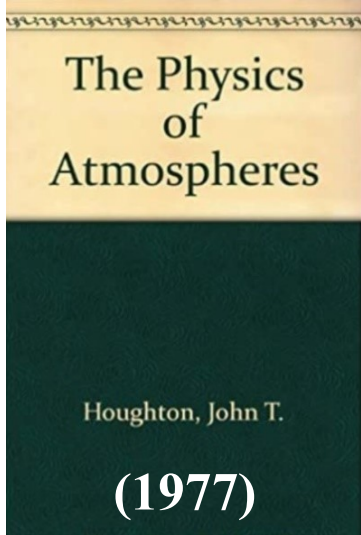
$$A - E = \frac{A_0}{2} \text{ independent of } \tau$$

Houghton (1977): $B_g - B_0 = \frac{\phi}{2\pi} \quad (2.13)$

$$B_g = \frac{\phi}{2\pi} (\chi_0^* + 2) \quad (2.15)$$

Goody and Yung (1989):

$$F/2\pi = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \quad (2.146)$$



The Physics
of
Atmospheres

Houghton, John T.

(1977)

2 (Houghton 1977, 2002)

A radiative equilibrium model

$$dI = -Ik\rho dz \quad (2.1)$$

where k is the absorption coefficient. Integrating (2.1) leads to

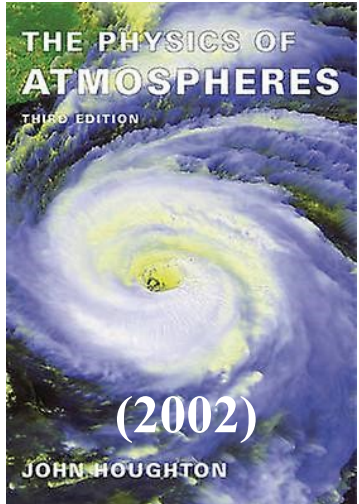
$$I = I_0 \exp\left(-\int k\rho dz\right) \quad (2.2)$$

The equation for radiative transfer through the slab, which includes both absorption and emission, is sometimes known as *Schwarzschild's equation*

$$dI = -Ik\rho dz + Bk\rho dz$$

or

$$\frac{dI}{d\chi} = I - B \quad (2.3)$$



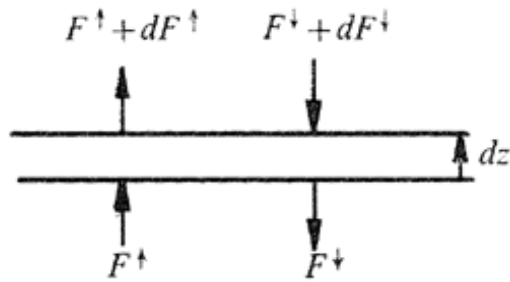


Fig. 2.3.

$$\frac{d}{dz}(F^{\downarrow} - F^{\uparrow}) = \rho c_p \frac{dT}{dt} \quad (2.4)$$

In equilibrium $dT/dt = 0$, and integration of (2.4) gives

$$F^{\uparrow} - F^{\downarrow} = \text{a constant } \phi, \text{ the net flux.} \quad (2.5)$$

Further, from (2.3) the transfer equations are

$$\left. \begin{aligned} \frac{dF^{\uparrow}}{d\chi^*} &= F^{\uparrow} - \pi B \\ -\frac{dF^{\downarrow}}{d\chi^*} &= F^{\downarrow} - \pi B \end{aligned} \right\} \quad (2.6)$$

If

$$\psi = F^\uparrow + F^\downarrow \quad (2.7)$$

equations (2.6) may be written

$$\frac{d\psi}{d\chi^*} = \phi \quad (2.8)$$

$$\frac{d\phi}{d\chi^*} = \psi - 2\pi B \quad (2.9)$$

Since $\phi = \text{constant}$ (from (2.5)) $d\phi/d\chi^* = 0$

and $\psi = 2\pi B \quad (2.10)$

$$B = \frac{\phi}{2\pi}\chi^* + \text{constant} \quad (2.11)$$

The boundary condition at the top of the atmosphere ($\chi^* = 0$) is $F^\downarrow = 0$, so that here $\psi = \phi$ and, from (2.10), the constant in (2.11) is $\phi/2\pi$, i.e.

$$B = \frac{\phi}{2\pi}(\chi^* + 1) \quad (2.12)$$

At the bottom of the atmosphere where $\chi^* = \chi_0^*$, $F^\uparrow = \pi B_g$, B_g being the black-body function at the temperature of the ground. It is easy to show that there must be a temperature discontinuity at the lower boundary, the black-body function for the air close to the ground being B_0 , and

$$B_g - B_0 = \frac{\phi}{2\pi} \quad (2.13)$$

Radiative equilibrium: discontinuity Radiative-convective equilibrium: Surface net radiation balanced by non-radiative fluxes, constrained unequivocally to OLR/2

Atmospheric Radiation

Theoretical Basis

SECOND EDITION

R. M. GOODY
and
Y. L. YUNG

Goody and Yung (1989)

$$-\frac{1}{e_{\nu,\nu}} \frac{dI_{\nu}(P, \mathbf{s})}{ds} = I_{\nu}(P, \mathbf{s}) - J_{\nu}(P, \mathbf{s}). \quad (2.17)$$

Equation (2.17) is known as the *equation of transfer*, and was first given in this form by Schwarzschild. While it sets the pattern of the formalism used in transfer problems, its physical content is very slight.

2.3.2. The general solution

The *optical path* along a ray trajectory from point 1 to point 2 in the direction \mathbf{s} is

$$\bar{\tau}(1, 2) = \int_1^2 e_{\nu} ds. \quad (2.84)$$

Note that we have defined $\bar{\tau}$ to be *positive definite*.

Consider the path of integration shown in Fig. 2.5. The equation of transfer at P' is

$$\frac{dI_{\nu}(P', \mathbf{s})}{d\bar{\tau}_{\nu}} = I_{\nu}(P', \mathbf{s}) - J_{\nu}(P', \mathbf{s}). \quad (2.85)$$

$$I_v(P, \mathbf{s}) = I_v(P'', \mathbf{s})e^{-\bar{\tau}_v(P'', P)} + \int_0^{\bar{\tau}_v(P'', P)} J_v(P', \mathbf{s})e^{-\bar{\tau}_v(P', P)} d\bar{\tau}_v. \quad (2.86)$$

$$I_v(P, \mathbf{s}) = \int_0^\infty J_v(P', \mathbf{s})e^{-\bar{\tau}_v} d\bar{\tau}_v. \quad (2.87)$$

2.3.3. Thermal radiation in a stratified atmosphere

We now consider an isotropic source function in an atmosphere for which absorption coefficient and temperature are functions of the vertical coordinate (z) alone (a *stratified atmosphere*, see Fig. 2.7).

The appropriate equation of radiative transfer in this case is, from (2.17) and Fig. 2.7,

$$-\frac{\xi}{e_{v,v}} \frac{dI_v(z, \xi)}{dz} = I_v(z, \xi) - J_v(z, \xi), \quad (2.91)$$

Substitute (2.142) and (2.143) in (2.133) and eliminate \bar{I} and either I^+ or I^- . There results at $\tau = \tau_1$ (lower boundary)

$$I^+(\tau_1) = F(\tau_1)/2\pi + B(\tau_1) + (1/4\pi)(dF/d\tau)_{\tau=\tau_1}, \quad (2.144)$$

and at $\tau = 0$ (upper boundary)

$$I^-(0) = -F(0)/2\pi + B(0) + (1/4\pi)(dF/d\tau)_{\tau=0}. \quad (2.145)$$

As an illustration, consider the case of radiative equilibrium with black bodies emitting $B^*(0)$ or $B^*(\tau_1)$ at the two boundaries. The third terms on the right-hand side of (2.144) and (2.145) are now zero and

$$F/2\pi = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \quad (2.146)$$

Equation (2.146) requires a discontinuity in the Planck function, implying a discontinuity of temperature, at the boundary.

Houghton (1977)

2.5 *The greenhouse effect*

Combining (2.12) and (2.13) we find that for the radiative equilibrium atmosphere

$$B_g = \frac{\phi}{2\pi} (\chi_0^* + 2) \quad (2.15)$$

where χ_0^* is the optical depth at the bottom of the atmosphere. If $\chi_0^* = 0$, $B_g = \phi/\pi$ and the surface temperature is in equilibrium with the incoming and the outgoing radiation, which are both equal to ϕ . If χ_0^* is large, the surface temperature represented by the black-body function B_g will be very considerably enhanced, an illustration of the so-called *greenhouse effect* mentioned in

$$\text{Clear-sky at } \chi^* = 2 \Rightarrow B_g = 2\phi/\pi$$

My all-sky versions

Eq. (1) Schwarzschild (1906, Eq. 11), Houghton (2.13), net, clear-sky:
 $A - E = \Delta A = A_0 / 2$

Eq. (2) Schwarzschild (1906, Eq. 11), incl LWCRE, net, all-sky:
 $A - E = \Delta A = (A_0 - L) / 2$

Eq. (3) Schwarzschild (1906, Eq. 11), at $\tau = 2$, Houghton (2.15), total, clear-sky:
 $A = 2A_0$

Eq. (4) Schwarzschild (1906, Eq. 11), at $\tau = 2$, incl LWCRE, total, all-sky:
 $A = 2A_0 + L$

DATA: The four equations

CERES EBAF Ed4.1, 240 months, Global Means, Dec 2000 – Nov 2020

	Wm^{-2}
Eq. (1) SFC (SW net + LW net) (clear) = OLR(clear)/2	- 2.29
Eq. (2) SFC (SW net + LW net) (all) = [OLR(all) – LWCRE]/2]	+ 2.77
Eq. (3) SFC (SW net + LW down) (clear) = 2OLR(clear)	- 2.82
Eq. (4) SFC (SW net + LW down) (all) = 2OLR(all) + LWCRE	+ 2.46
Mean bias:	- 0.03

Surface SW net is NOT resolved into its downward and upward components

Theory: Global mean energy budget components

$$\text{Eq. (1)} \quad \underbrace{\text{Surface SW net(clear)}}_{x_1} + \underbrace{\text{LW down(clear)}}_{x_2} - \underbrace{\text{LW up(clear)}}_{x_3} = \underbrace{\text{OLR(clear)}}_{x_4} / 2$$

$$\text{Eq. (1)} \quad x_1 + x_2 - x_3 = x_4/2$$

$$\text{Eq. (2)} \quad \underbrace{\text{Surface SW net(all)}}_{x_5} + \text{LW down(all)} - \text{LW up(all)} = [\text{OLR(all)} - \text{LWCRE}] / 2$$

Definitions:

Surface LW down (all)	= Surface LW down(clear) + LWCRE
Surface LW up (all)	= Surface LW up (clear)
OLR (all)	= OLR(clear) - LWCRE

Let be LWCRE at TOA = LWCRE at the surface = 1

$$\text{Eq. (2)} \quad x_5 + x_2 + 1 - x_3 = (x_4 - 2)/2 = x_4/2 - 1$$

Eq. (3) Surface SW net(clear) + LW down(clear) = 2OLR(clear)

$$x_1 + x_2 = 2x_4 \Rightarrow x_3 = 3x_4/2$$

Eq. (4) Surface SW net(all) + LW down(all) = 2OLR(all) + LWCRE
= 2OLR(clear) – LWCRE

$$x_5 + x_2 + 1 = 2x_4 - 1$$

From now, unit = LWCRE = 1

Integer solution

$$\text{LWCRE at surface} = \text{LWCRE at TOA} = \mathbf{1}$$

$$\mathbf{X}_1 = \text{Surface SW net (clear-sky)} = \mathbf{8}$$

$$\mathbf{X}_2 = \text{Surface LW down (clear-sky)} = \mathbf{12}$$

$$\mathbf{X}_3 = \text{Surface LW up (all-sky, clear-sky)} = \mathbf{15}$$

$$\mathbf{X}_4 = \text{TOA LW up (clear-sky)} = \mathbf{10}$$

$$\mathbf{X}_5 = \text{Surface SW net (all-sky)} = \mathbf{6}$$

$$\text{TOA LW up (OLR) (all-sky)} = \mathbf{9}$$

$$\text{Surface LW down (DLR) (all-sky)} = \mathbf{13}$$

Greenhouse effect:

$$\text{G(all)} : \text{Surface LW up} - \text{TOA LW up (all)} = \mathbf{15} - \mathbf{9} = \mathbf{6}.$$

$$\text{G(clear)} : \text{Surface LW up} - \text{TOA LW up (clear)} = \mathbf{15} - \mathbf{10} = \mathbf{5}.$$

Integer solution best fit

Surface LW up, all-sky	=	15	Surface LW up, clear-sky	=	15
Surface SW net, all-sky	=	6	Surface SW net, clear-sky	=	8
Surface LW net, all-sky	=	-2	Surface LW net, clear-sky	=	-3
Surface SW+LW net, all-sky	=	4	Surface SW+LW net, clr-sky	=	5
Surface SW+LW gross, all	=	19	Surface SW+LW gross, clear	=	20
Surface LW down, all-sky	=	13	Surface LW down, clear-sky	=	12
TOA LW all-sky	=	9	TOA LW clear-sky	=	10
G greenhouse effect, all-sky	=	6	G greenhouse effect, clear-sky	=	5
LWCRE (surface, TOA)	=	1	SWCRE (surface)	=	-2

CERES EBAF Ed4.1, 240 months, Dec 2000 — Nov 2020 data, best fit:

LWCRE = 1 unit = 1 = 26.68 ± 0.01 Wm⁻².

$$\text{Eq. (1) SFC SW net} + \text{LW down} - \text{LW up (clear)} = (\text{TOA LW up, clear}) / 2$$

CERES 20-year clear-sky (with Δ^c)	Data	N × UNIT	Theory	Diff
SFC SW net	211.74	8 × 26.68	213.44	-1.70
SFC LW down	317.51	12 × 26.68	320.16	-2.65
SFC LW up	398.53	15 × 26.68	400.20	-1.67
TOA LW up	266.04	10 × 26.68	266.80	-0.76
SW+LW net	130.72	5 × 26.68	133.40	-2.68
G	132.49	5 × 26.68	133.40	-0.91

$$\text{Eq. (1) } \mathbf{8 + 12 - 15} = \mathbf{5} = \mathbf{10/2} \quad \mathbf{-2.29}$$

$$\text{Eq. (2) SFC SW net} + \text{LW down} - \text{LW up (all)} = [\text{TOA LW up (all)} - \text{LWCRE}] / 2$$

CERES 20-year all-sky	Data	N × UNIT	Theory	Diff
SFC SW net	163.63	6 × 26.68	160.08	3.55
SFC LW down	345.13	13 × 26.68	346.84	-1.71
SFC LW up	398.75	15 × 26.68	400.20	-1.45
TOA LW up	240.26	9 × 26.68	240.12	0.14
LWCRE	25.78	1 × 26.68	26.68	-0.90
SW+LW net	110.01	4 × 26.68	106.72	3.29

$$\text{Eq. (2) } \mathbf{6 + 13 - 15} = \mathbf{4} = \mathbf{(9 - 1)/2} \quad \mathbf{2.77}$$

Mean bias of the four equations

CERES EBAF Ed4.1, Dec 2000 – Nov 2020

• Net (clear-sky)	ΔEq1	= -2.29	}	0.24
• Net (all-sky)	ΔEq2	= 2.77		
• Gross (clear-sky)	ΔEq3	= -2.82	}	-0.18
• Gross (all-sky)	ΔEq4	= 2.46		
				mean = 0.03 Wm⁻²

• Clear-sky (net)	ΔEq1	= -2.29	}	-2.55
• Clear-sky (gross)	ΔEq3	= -2.82		
• All-sky (net)	ΔEq2	= 2.77	}	2.61
• All-sky (gross)	ΔEq4	= 2.46		
				mean = 0.03 Wm⁻²

DATA: Greenhouse Effect

CERES 12 months, Dec 2019 – Nov 2020

THEORY: $g(\text{clear})$

Eq.(1) $B_g - B_0 = B_{\text{eff}}/2$

Eq.(3) $B_g = 2B_{\text{eff}}$

$B_G = B_0 - B_{\text{eff}} \Rightarrow$

$B_G : B_{\text{eff}} : B_0 : B_g =$

$1 : 2 : 3 : 4 =$

5 : 10 : 15 : 20

$G : \text{OLR} : \text{ULW} =$

5 : 10 : 15 =

$1 : 2 : 3 \Rightarrow$

$g = G/\text{ULW} = 1/3$

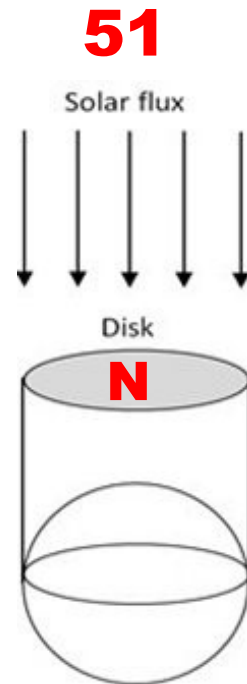
1 = 26.68 Wm⁻²

Month	OLR(clear)	ULW	G(clear)	g(clear)
Dec 2019	263.904	391.744	127.84	0.3263
Jan 2020	263.279	390.911	127.632	0.3265
Feb 2020	264.479	392.926	128.447	0.3269
Mar 2020	264.877	396.008	131.131	0.3311
Apr 2020	265.558	400.41	134.852	0.3368
May 2020	267.679	404.095	136.416	0.3376
Jun 2020	269.113	407.299	138.186	0.3393
Jul 2020	270.143	408.587	138.444	0.3388
Aug 2020	270.527	407.728	137.201	0.3365
Sep 2020	268.54	405.008	136.468	0.3370
Oct 2020	266.475	399.627	133.152	0.3332
Nov 2020	264.73	394.438	129.709	0.3288
Data	266.61	399.90	133.29	0.3333
Theory	266.80	400.20	133.40	0.3333

DATA: TOA SW up is integer

CERES EBAF Ed4.1, Dec 2000 – Nov 2020

TOA Flux (clear-sky with Δ^c)	N	Data	Theory	Diff
<i>SW up clear-sky</i>	8 / 4	53.72	53.36	0.36
<i>SW up all-sky</i>	15 / 4	98.97	100.05	-1.08
LW up clear-sky	40 / 4	266.04	266.80	-0.76
LW up all-sky	36 / 4	240.26	240.12	0.14
TOA SW CRE	-7 / 4	-45.25	-46.69	1.44
TOA LW CRE	4 / 4	25.78	26.68	-0.90
TOA Net CRE	-3 / 4	-19.47	-20.01	0.54
Albedo, clear	8 / 51	0.158	0.157	0.001
Albedo, all	15 / 51	0.291	0.294	-0.003



Clear-sky: SW up = **8** SW in = **43** LW up = **40** TOA Net IMB = **3**

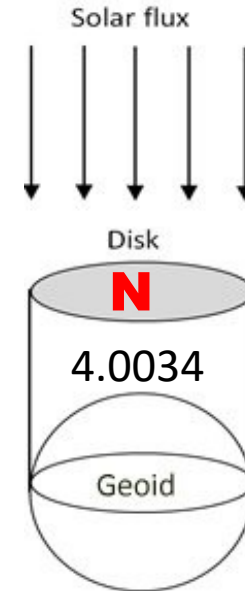
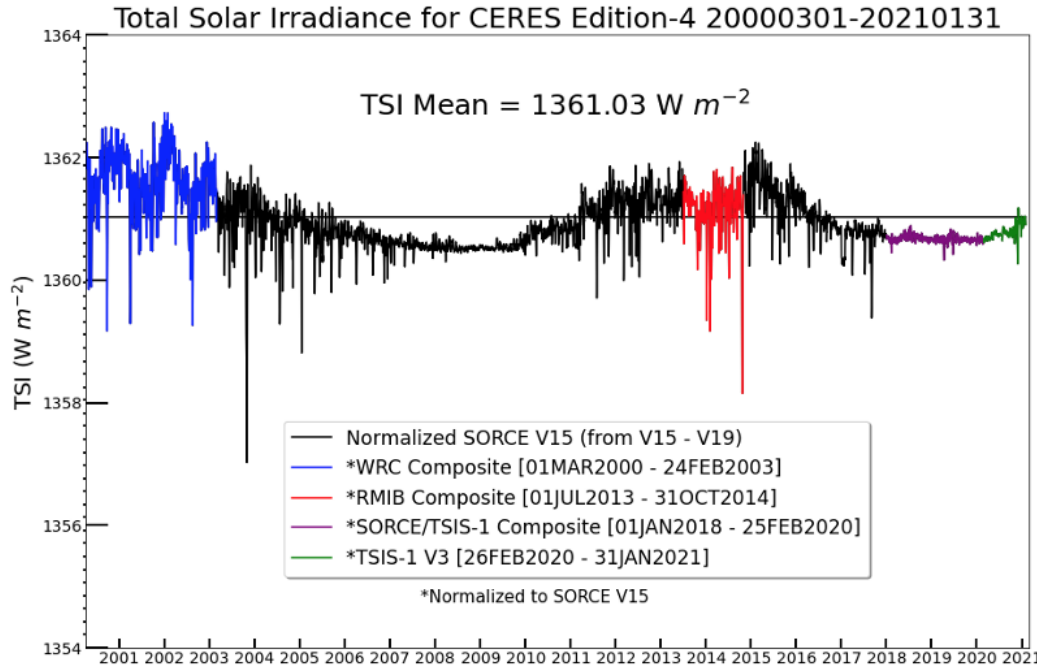
All-sky: SW up = **15** SW in = **36** LW up = **36** TOA Net CRE = **-3**

With TSI = **51**, each flux is an **integer** on the intercepting cross-section disk.

Data: TSI

TSI = **51** => **1** = LWCRE (spherical weighting)

TSI = **51** => UNIT = (TSI/51) × 4/4.0034 (geodetic weighting)



TSI = 1361.03 => **1** = LWCRE = 26.69 (spherical weighting)

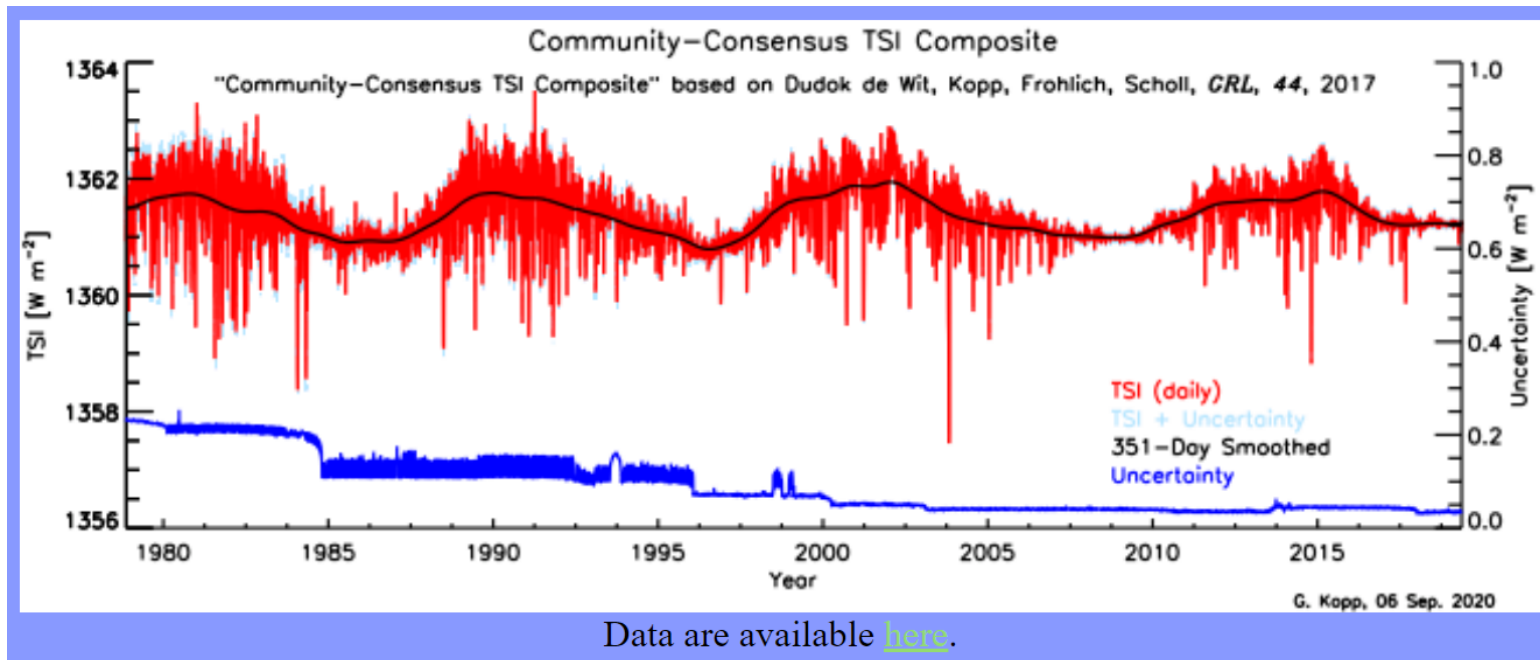
TSI = 1361.03 => UNIT = **1** = 26.67 (geodetic weighting) SORCE

340.020

7647	1361.311
7648	1361.303
7649	1361.333
7650	1361.327
7651	1361.364
7652	1361.36
7653	1361.356
7654	1361.352
7655	1361.348
7656	1361.344
7657	1361.34
7658	1361.336
7659	1361.301
7660	1361.33
7661	1361.358
7662	1361.397
7663	1361.383
7664	1361.318
7665	1361.189
7666	1361.091
7667	1361.096
7668	1361.133
7669	1361.261
7670	1361.344
7671	1361.413
7672	1360.993

Greg's TSI Page

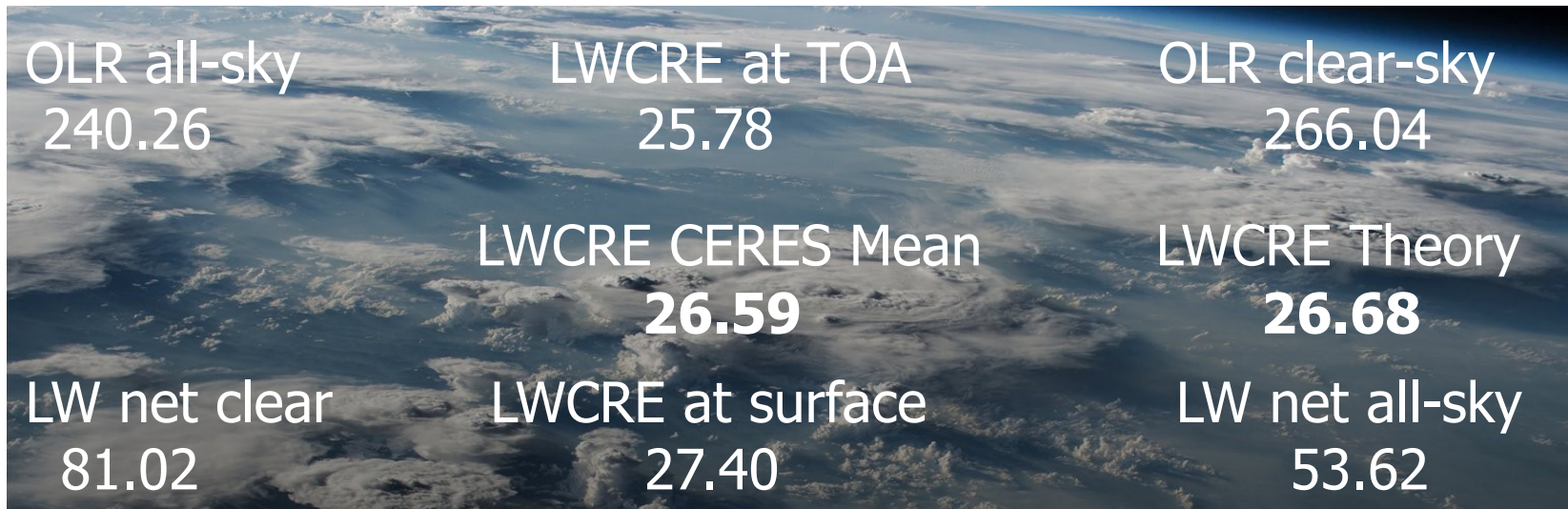
TSI composite created by G. Kopp on 4 Sept. 2020



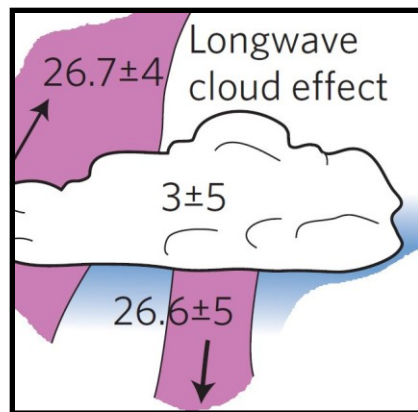
$$(1361.63/51) \times (4/4.0034) = \mathbf{1} = \mathbf{26.68 \pm 0.01 \text{ Wm}^{-2}}$$

14767	1361.456
14768	1361.457
14769	1361.428
14770	1361.44
14771	1361.447
14772	1361.421
14773	1361.439
14774	1361.42
14775	1361.411
14776	1361.392
14777	1361.391
14778	1361.454
14779	1361.445
14780	1361.475
14781	1361.488
14782	1361.445
14783	1361.434
14784	1361.41
14785	1361.363
14786	1361.343
14787	1361.335
14788	1361.381
14789	1361.417
14790	1361.506
14791	1361.522
14792	1361.63

The Greenhouse Effect of Clouds: $\Delta F(\text{CERES}) = -0.09 \text{ Wm}^{-2}$

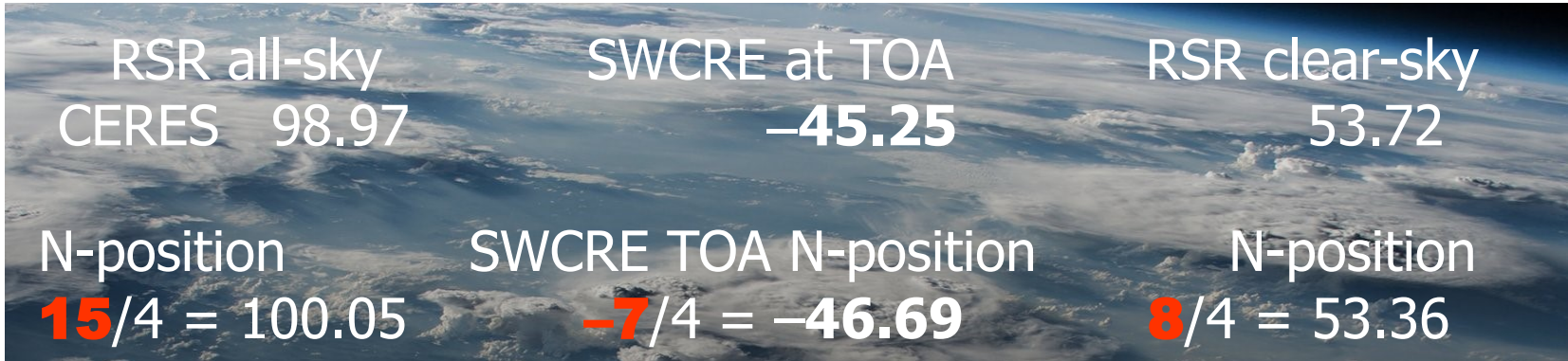


Stephens
et al. (2012)
LWCRE mean
= 26.65 Wm^{-2}
 $\Delta F(\text{Stephens}) =$
 -0.03 Wm^{-2}



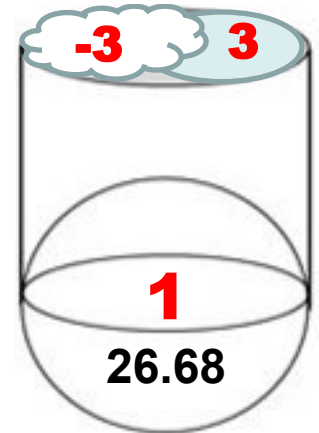
LWCRE Theory
TSI = 51
1 = 26.68 Wm^{-2}
4 Eqs 20-yr
1 = 26.68 Wm^{-2}

The Shielding Effect of Clouds: $\Delta F(\text{CERES}) = 1.44 \text{ Wm}^{-2}$

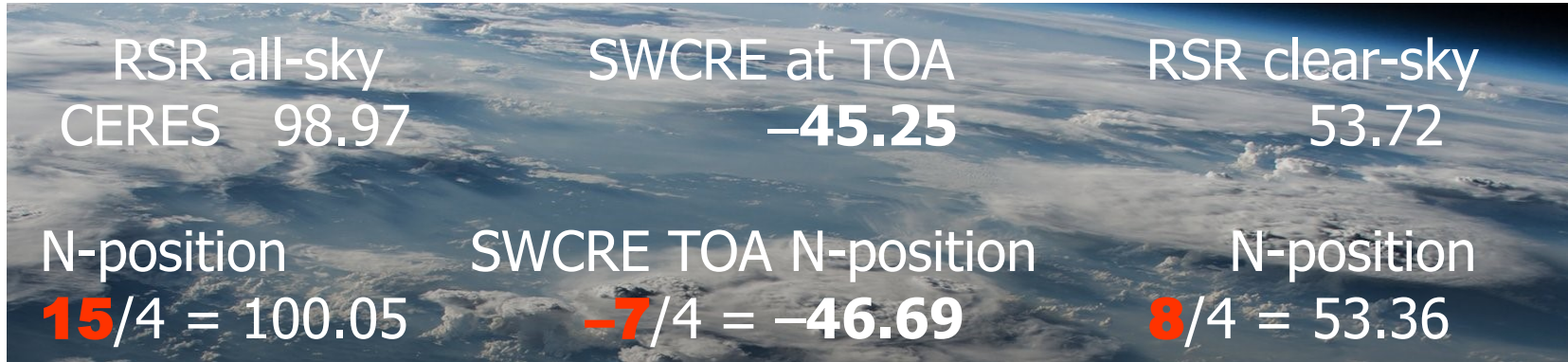


$$\begin{aligned}
 \text{TOA Net CRE} &= \text{LWCRE} + \text{SWCRE} = \\
 &= \text{OLR}(\text{clear}) - \text{OLR}(\text{all}) + \text{RSR}(\text{clear}) - \text{RSR}(\text{all}) = \\
 &= \text{OLR}(\text{clear}) - \text{OLR}(\text{all}) + \text{ISR} - \text{ASR}(\text{clear}) - [\text{ISR} - \text{ASR}(\text{all})] = \\
 &= [\text{ASR}(\text{all}) - \text{OLR}(\text{all})] - [(\text{ASR}(\text{clear}) - \text{OLR}(\text{clear}))] = \\
 &= \text{TOA Net IMB}(\text{all}) - \text{TOA Net IMB}(\text{clear})
 \end{aligned}$$

TOA Net CRE TOA IMB(clear)



The Shielding Effect of Clouds: $\Delta F(\text{CERES}) = 1.44 \text{ Wm}^{-2}$



=> **EEI = TOA Net CRE + TOA Net IMB (clear)**

DATA	25.78 - 45.25	= TOA Net CRE	= -19.47 Wm^{-2}
	340.01 - 53.72 - 266.04	= TOA Net IMB (clear)	= 20.25 Wm^{-2}
	EEI	= TOA Net IMB (all)	= 0.78 Wm^{-2}

THEORY	TOA Net CRE	= 4 /4 + 8 /4 - 15 /4 = -3 /4 = -20.01 Wm^{-2}
	TOA Net IMB (clear)	= 51 /4 - 8 /4 - 40 /4 = 3 /4 = 20.01 Wm^{-2}

