Data versus theory update

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CERES Science Team Meeting, May 13, 2021
Data
CERES EBAF Ed4.1, 240 months (Dec 2000 – Nov 2020)

Theory
Schwarzschild (1906, Eq. 11)
Theory

On the Equilibrium of the Sun’s Atmosphere

Schwarzschild (1906)

Radiative equilibrium

Two-stream

A upward beam

B downward beam

E emission of the layer

Ueber das Gleichgewicht der Sonnenatmosphäre

Von

K. Schwarzschild.


c) Radiative Equilibrium. If we assume that the outer regions of the sun

Assume that each layer \( dh \) of the solar atmosphere absorbs a fraction \( \alpha dh \) of the transmitted radiation. If \( E \) is the emission of a black body at the temperature of this layer \( dh \), and assuming that Kirchhoff’s law applies, it follows that this layer radiates the energy \( aEdh \) in every direction.

Consider now, at some point in the solar atmosphere, the radiative energy \( A \) which is transmitted outward, and the radiative energy \( B \), which (due to the radiation of outer layers) is transmitted inward.

Treat first the inward energy \( B \). When traveling inward through an infinitesimally thin layer \( dh \), the fraction \( \alpha Bdh \) of \( B \) will be lost; on the other hand, the contribution \( aEdh \) due to the lateral radiation of the layer itself will be added to \( B \). All in all,

\[
\frac{dB}{dh} = a(E - B).
\]
K. Schwarzschild

In the case of the outward energy $A$, we proceed analogously and obtain

$$\frac{dA}{dh} = -a(E - A). \quad (8)$$

Given the absorption coefficient $a$ as a function of depth $h$, define the “average optical depth”\* of the atmosphere lying above the depth $h$ by

$$\bar{\tau} = \int_{0}^{h} adh. \quad (9)$$

The differential equations then become

$$\frac{dB}{d\bar{\tau}} = E - B, \quad \frac{dA}{d\bar{\tau}} = A - E. \quad (10)$$

We want to find the temperature distribution under steady-state conditions. These require that each layer receives as much energy as it radiates, i.e., that

$$aA + aB = 2aE, \quad A + B = 2E.$$

Introducing the parameter $\zeta$ such that

$$A = E + \zeta, \quad B = E - \zeta,$$
we obtain the differential equations in the form

\[ \frac{d\zeta}{d\bar{\tau}} = 0, \quad \frac{dE}{d\bar{\tau}} = \zeta, \]

and after integration we have

\[ \zeta = \text{const.} \]
\[ E = E_0 + \zeta \bar{\tau} \]
\[ A = E_0 + \zeta (1 + \bar{\tau}) \]
\[ B = E_0 + \zeta (\bar{\tau} - 1). \]

The constants of integration $E_0$ and $\zeta$ are fixed by the requirements that there can be no inward radiation at the outer boundary of the atmosphere ($\bar{\tau} = 0$), and that the outward energy there must have the observed value $A$. Thus at $\bar{\tau} = 0$ we must have

\[ B = 0, \quad A = A_0. \]

This leads to the final result

\[ E = \frac{A_0}{2} (1 + \bar{\tau}), \quad A = \frac{A_0}{2} (2 + \bar{\tau}), \quad B = \frac{A_0}{2} \bar{\tau}. \]
\[ E = \frac{A_0}{2} (1 + \tau), \quad A = \frac{A_0}{2} (2 + \tau), \quad B = \frac{A_0}{2} \tau. \quad (11) \]

\[ A - E = \frac{A_0}{2} \text{ independent of } \tau \]

Houghton (1977): \[ B_g - B_0 = \frac{\phi}{2\pi} \quad (2.13) \]

\[ B_g = \frac{\phi}{2\pi} (\chi_0^* + 2) \quad (2.15) \]

Goody and Yung (1989):

\[ \frac{F}{2\pi} = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \quad (2.146) \]
A radiative equilibrium model

\[ dI = -I k \rho \, dz \]  \hspace{1cm} (2.1)

where \( k \) is the absorption coefficient. Integrating (2.1) leads to

\[ I = I_0 \exp \left( - \int k \rho \, dz \right) \] \hspace{1cm} (2.2)

The equation for radiative transfer through the slab, which includes both absorption and emission, is sometimes known as Schwarzchild’s equation

\[ dI = -I k \rho \, dz + B k \rho \, dz \]

or \[ \frac{dI}{d\chi} = I - B \] \hspace{1cm} (2.3)
\[
\frac{d}{dz} (F_{\uparrow} - F_{\downarrow}) = \rho c_p \frac{dT}{dt}
\] (2.4)

In equilibrium \(dT/dt = 0\), and integration of (2.4) gives

\[F_{\uparrow} - F_{\downarrow} = \text{a constant } \phi, \text{ the net flux.} \] (2.5)

Further, from (2.3) the transfer equations are

\[
\begin{align*}
\frac{dF_{\uparrow}}{d\chi^*} &= F_{\uparrow} - \pi B \\
-\frac{dF_{\downarrow}}{d\chi^*} &= F_{\downarrow} - \pi B
\end{align*}
\] (2.6)
If
\[ \psi = F^\uparrow + F^\downarrow \]  

(2.7)
equations (2.6) may be written
\[ \frac{d\psi}{d\chi^*} = \phi \]  

(2.8)
\[ \frac{d\phi}{d\chi^*} = \psi - 2\pi B \]  

(2.9)
Since \( \phi = \) constant (from (2.5)) \( \frac{d\phi}{d\chi^*} = 0 \)
and \[ \psi = 2\pi B \]  

(2.10)
Radiative equilibrium: discontinuity

\[ B = \frac{\phi}{2\pi} \chi^* + \text{constant} \quad (2.11) \]

The boundary condition at the top of the atmosphere (\( \chi^* = 0 \)) is \( F^\downarrow = 0 \), so that here \( \psi = \phi \) and, from (2.10), the constant in (2.11) is \( \phi/2\pi \), i.e.

\[ B = \frac{\phi}{2\pi} (\chi^* + 1) \quad (2.12) \]

At the bottom of the atmosphere where \( \chi^* = \chi_0^* \), \( F^\uparrow = \pi B_g \), \( B_g \) being the black-body function at the temperature of the ground. It is easy to show that there must be a temperature discontinuity at the lower boundary, the black-body function for the air close to the ground being \( B_0 \), and

\[ B_g - B_0 = \frac{\phi}{2\pi} \]

Radiative equilibrium: discontinuity
Radiative-convective equilibrium:
Surface net radiation balanced by non-radiative fluxes, constrained unequivocally to OLR/2

\[ (2.13) \]
Goody and Yung (1989)

\[- \frac{1}{e_{v,v}} \frac{dI_v(P, s)}{ds} = I_v(P, s) - J_v(P, s). \] (2.17)

Equation (2.17) is known as the *equation of transfer*, and was first given in this form by Schwarzschild. While it sets the pattern of the formalism used in transfer problems, its physical content is very slight.

2.3.2. The general solution

The *optical path* along a ray trajectory from point 1 to point 2 in the direction $s$ is

\[ \bar{\tau}(1, 2) = \int_{1}^{2} e_v \, ds. \] (2.84)

Note that we have defined $\bar{\tau}$ to be *positive definite*.

Consider the path of integration shown in Fig. 2.5. The equation of transfer at $P'$ is

\[ \frac{dI_v(P', s)}{d\bar{\tau}_v} = I_v(P', s) - J_v(P', s). \] (2.85)
\[ I_\nu(P, s) = I_\nu(P'', s)e^{-\tau_\nu(P'', P)} + \int_0^{\tau_\nu(P'', P)} J_\nu(P', s)e^{-\tau_\nu(P', P)} d\tau_\nu. \quad (2.86) \]

\[ I_\nu(P, s) = \int_0^{\infty} J_\nu(P', s)e^{-\tau_\nu} d\tau_\nu. \quad (2.87) \]

2.3.3. Thermal radiation in a stratified atmosphere

We now consider an isotropic source function in an atmosphere for which absorption coefficient and temperature are functions of the vertical coordinate \( z \) alone (a stratified atmosphere, see Fig. 2.7).

The appropriate equation of radiative transfer in this case is, from (2.17) and Fig. 2.7,

\[ -\frac{\xi}{e_{v,v}} \frac{dI_\nu(z, \xi)}{dz} = I_\nu(z, \xi) - J_\nu(z, \xi), \quad (2.91) \]
Substitute (2.142) and (2.143) in (2.133) and eliminate \( \tilde{I} \) and either \( I^+ \) or \( I^- \). There results at \( \tau = \tau_1 \) (lower boundary)

\[
I^+(\tau_1) = F(\tau_1)/2\pi + B(\tau_1) + (1/4\pi)(dF/d\tau)_{\tau=\tau_1}, \tag{2.144}
\]

and at \( \tau = 0 \) (upper boundary)

\[
I^-(0) = -F(0)/2\pi + B(0) + (1/4\pi)(dF/d\tau)_{\tau=0}. \tag{2.145}
\]

As an illustration, consider the case of radiative equilibrium with black bodies emitting \( B^*(0) \) or \( B^*(\tau_1) \) at the two boundaries. The third terms on the right-hand side of (2.144) and (2.145) are now zero and

\[
F/2\pi = B(0) - B^*(0) = B^*(\tau_1) - B(\tau_1). \tag{2.146}
\]

Equation (2.146) requires a discontinuity in the Planck function, implying a discontinuity of temperature, at the boundary.
2.5 The greenhouse effect

Combining (2.12) and (2.13) we find that for the radiative equilibrium atmosphere

\[ B_g = \frac{\phi}{2\pi} (\chi_0^* + 2) \]  

(2.15)

where \( \chi_0^* \) is the optical depth at the bottom of the atmosphere. If \( \chi_0^* = 0 \), \( B_g = \phi/\pi \) and the surface temperature is in equilibrium with the incoming and the outgoing radiation, which are both equal to \( \phi \). If \( \chi_0^* \) is large, the surface temperature represented by the black-body function \( B_g \) will be very considerably enhanced, an illustration of the so-called greenhouse effect mentioned in

Clear-sky at \( \chi^* = 2 \implies B_g = 2\phi/\pi \)
My all-sky versions

Eq. (1)  \[ A - E = \Delta A = A_0 / 2 \]  
Schwarzschild (1906, Eq. 11), Houghton (2.13), net, clear-sky:

Eq. (2)  \[ A - E = \Delta A = (A_0 - L) / 2 \]  
Schwarzschild (1906, Eq. 11), incl LWCRE, net, all-sky:

Eq. (3)  \[ A = 2A_0 \]  
Schwarzschild (1906, Eq. 11), at \( \tau = 2 \), Houghton (2.15), total, clear-sky:

Eq. (4)  \[ A = 2A_0 + L \]  
Schwarzschild (1906, Eq. 11), at \( \tau = 2 \), incl LWCRE, total, all-sky:
DATA: The four equations
CERES EBAF Ed4.1, 240 months, Global Means, Dec 2000 – Nov 2020

Eq. (1) SFC (SW net + LW net) (clear) = OLR(clear)/2

\[-2.29\]

Eq. (2) SFC (SW net + LW net) (all) = [OLR(all) – LWCRE)/2]

\[+2.77\]

Eq. (3) SFC (SW net + LW down) (clear) = 2OLR(clear)

\[-2.82\]

Eq. (4) SFC (SW net + LW down) (all) = 2OLR(all) + LWCRE

\[+2.46\]

Mean bias:

\[-0.03\]

Surface SW net is NOT resolved into its downward and upward components
Theory: Global mean energy budget components

**Eq. (1)**
Surface SW net(clear) + LW down(clear) - LW up(clear) = OLR(clear) / 2

\[
x_1 + x_2 - x_3 = x_4/2
\]

**Eq. (2)**
Surface SW net(all) + LW down(all) - LW up(all) = [OLR(all) - LWCRE] / 2

\[
x_5 + x_2 + 1 - x_3 = (x_4 - 2)/2 = x_4/2 - 1
\]

Definitions:
- Surface LW down (all) = Surface LW down(clear) + LWCRE
- Surface LW up (all) = Surface LW up (clear)
- OLR (all) = OLR(clear) - LWCRE

Let be LWCRE at TOA = LWCRE at the surface = 1
**Eq. (3)** Surface SW net(clear) + LW down(clear) = 2OLR(clear)

\[ x_1 + x_2 = 2x_4 \Rightarrow x_3 = 3x_4 / 2 \]

**Eq. (4)** Surface SW net(all) + LW down(all) = 2OLR(all) + LWCRE = 2OLR(clear) – LWCRE

\[ x_5 + x_2 + 1 = 2x_4 - 1 \]

From now, unit = LWCRE = 1
Integer solution

LWCRE at surface = LWCRE at TOA = 1

\(X_1 = \) Surface SW net (clear-sky) = 8
\(X_2 = \) Surface LW down (clear-sky) = 12
\(X_3 = \) Surface LW up (all-sky, clear-sky) = 15
\(X_4 = \) TOA LW up (clear-sky) = 10
\(X_5 = \) Surface SW net (all-sky) = 6

TOA LW up (OLR) (all-sky) = 9
Surface LW down (DLR) (all-sky) = 13

Greenhouse effect:
\(G(\text{all}) : \) Surface LW up – TOA LW up (all) = 15 – 9 = 6.
\(G(\text{clear}) : \) Surface LW up – TOA LW up (clear) = 15 – 10 = 5.
Integer solution best fit

Surface LW up, all-sky  = 15  
Surface SW net, all-sky  = 6  
Surface LW net, all-sky  = -2  
Surface SW+LW net, all-sky  = 4  
Surface SW+LW gross, all  = 19  
Surface LW down, all-sky  = 13  
TOA LW all-sky  = 9  
G greenhouse effect, all-sky  = 6  
LWCRE (surface, TOA)  = 1

Surface LW up, clear-sky  = 15  
Surface SW net, clear-sky  = 8  
Surface LW net, clear-sky  = -3  
Surface SW+LW net, clr-sky  = 5  
Surface SW+LW gross, clear  = 20  
Surface LW down, clear-sky  = 12  
TOA LW clear-sky  = 10  
G greenhouse effect, clear-sky  = 5  
SWCRE (surface)  = -2

CERES EBAF Ed4.1, 240 months, Dec 2000 — Nov 2020 data, best fit:

LWCRE = 1 unit = 1 = 26.68 ± 0.01 Wm⁻².
Eq. (1) SFC SW net + LW down – LW up (clear) = (TOA LW up, clear) / 2

<table>
<thead>
<tr>
<th>CERES 20-year clear-sky (with $\Delta^c$)</th>
<th>Data</th>
<th>$N \times$ UNIT</th>
<th>Theory</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFC SW net</td>
<td>211.74</td>
<td>$8 \times 26.68$</td>
<td>213.44</td>
<td>$-1.70$</td>
</tr>
<tr>
<td>SFC LW down</td>
<td>317.51</td>
<td>$12 \times 26.68$</td>
<td>320.16</td>
<td>$-2.65$</td>
</tr>
<tr>
<td>SFC LW up</td>
<td>398.53</td>
<td>$15 \times 26.68$</td>
<td>400.20</td>
<td>$-1.67$</td>
</tr>
<tr>
<td>TOA LW up</td>
<td>266.04</td>
<td>$10 \times 26.68$</td>
<td>266.80</td>
<td>$-0.76$</td>
</tr>
<tr>
<td>SW+LW net</td>
<td>130.72</td>
<td>$5 \times 26.68$</td>
<td>133.40</td>
<td>$-2.68$</td>
</tr>
<tr>
<td>G</td>
<td>132.49</td>
<td>$5 \times 26.68$</td>
<td>133.40</td>
<td>$-0.91$</td>
</tr>
</tbody>
</table>

Eq. (1) $8 + 12 - 15 = 5 = 10/2$ $-2.29$
Eq. (2) SFC SW net + LW down – LW up (all) = \left[ \text{TOA LW up (all)} – \text{LWCRE} \right] / 2

<table>
<thead>
<tr>
<th>CERES 20-year all-sky</th>
<th>Data</th>
<th>N × UNIT</th>
<th>Theory</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFC SW net</td>
<td>163.63</td>
<td>\textbf{6} \times 26.68</td>
<td>160.08</td>
<td>3.55</td>
</tr>
<tr>
<td>SFC LW down</td>
<td>345.13</td>
<td>\textbf{13} \times 26.68</td>
<td>346.84</td>
<td>–1.71</td>
</tr>
<tr>
<td>SFC LW up</td>
<td>398.75</td>
<td>\textbf{15} \times 26.68</td>
<td>400.20</td>
<td>–1.45</td>
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<tr>
<td>TOA LW up</td>
<td>240.26</td>
<td>\textbf{9} \times 26.68</td>
<td>240.12</td>
<td>0.14</td>
</tr>
<tr>
<td>LWCRE</td>
<td>25.78</td>
<td>\textbf{1} \times 26.68</td>
<td>26.68</td>
<td>–0.90</td>
</tr>
<tr>
<td>SW+LW net</td>
<td>110.01</td>
<td>\textbf{4} \times 26.68</td>
<td>106.72</td>
<td>3.29</td>
</tr>
</tbody>
</table>

Eq. (2) \textbf{6} + \textbf{13} – \textbf{15} = \textbf{4} = (9 – 1)/2 = 2.77
Eq. (3) SFC SW net + LW down (clear) = 2 × TOA LW up (clear)

<table>
<thead>
<tr>
<th>CERES 20-year clear-sky (with $\Delta^c$)</th>
<th>Data</th>
<th>$N \times \text{UNIT}$</th>
<th>Theory</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFC SW net</td>
<td>211.74</td>
<td>8 × 26.68</td>
<td>213.44</td>
<td>−1.70</td>
</tr>
<tr>
<td>SFC LW down</td>
<td>317.51</td>
<td>12 × 26.68</td>
<td>320.16</td>
<td>−2.65</td>
</tr>
<tr>
<td>SW net + LW down</td>
<td>529.25</td>
<td>20 × 26.68</td>
<td>533.60</td>
<td>−4.35</td>
</tr>
<tr>
<td>TOA LW up</td>
<td>266.04</td>
<td>10 × 26.68</td>
<td>266.80</td>
<td>−0.76</td>
</tr>
</tbody>
</table>

Eq. (3) 8 + 12 = 20 = 2 × 10

$-2.82$
Eq. (4) \[ SFC \text{ SW net} + \text{LW down} \ (\text{all}) = 2 \times \text{TOA LW up} \ (\text{all}) + \text{LWCRE} \]

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>(\text{N} \times \text{UNIT})</th>
<th>Theory</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFC SW net</td>
<td>163.63</td>
<td>6 \times 26.68</td>
<td>160.08</td>
<td>3.55</td>
</tr>
<tr>
<td>SFC LW down</td>
<td>345.13</td>
<td>13 \times 26.68</td>
<td>346.84</td>
<td>-1.71</td>
</tr>
<tr>
<td>TOA LW up</td>
<td>240.26</td>
<td>9 \times 26.68</td>
<td>240.12</td>
<td>0.14</td>
</tr>
<tr>
<td>LWCRE</td>
<td>25.78</td>
<td>1 \times 26.68</td>
<td>26.68</td>
<td>-0.90</td>
</tr>
<tr>
<td>SW net + LW down</td>
<td>508.76</td>
<td>19 \times 26.68</td>
<td>506.92</td>
<td>1.84</td>
</tr>
<tr>
<td>2OLR + LWCRE</td>
<td>506.30</td>
<td>19 \times 26.68</td>
<td>506.92</td>
<td>-0.62</td>
</tr>
</tbody>
</table>

Eq. (4) \[ 6 + 13 = 19 = 2 \times 9 + 1 \]

2.46
Mean bias of the four equations
CERES EBAF Ed4.1, Dec 2000 – Nov 2020

- Net (clear-sky) \[ \Delta \text{Eq1} = -2.29 \] \{ 0.24 \\
- Net (all-sky) \[ \Delta \text{Eq2} = 2.77 \] \{ 0.24 \\
- Gross (clear-sky) \[ \Delta \text{Eq3} = -2.82 \] \{ -0.18 \\
- Gross (all-sky) \[ \Delta \text{Eq4} = 2.46 \] \{ -0.18 \\

mean = 0.03 Wm\(^{-2}\)

- Clear-sky (net) \[ \Delta \text{Eq1} = -2.29 \] \{ -2.55 \\
- Clear-sky (gross) \[ \Delta \text{Eq3} = -2.82 \] \{ -2.55 \\
- All-sky (net) \[ \Delta \text{Eq2} = 2.77 \] \{ 2.61 \\
- All-sky (gross) \[ \Delta \text{Eq4} = 2.46 \] \{ 2.61 \\

mean = 0.03 Wm\(^{-2}\)
**DATA: Greenhouse Effect**

**CERES 12 months, Dec 2019 – Nov 2020**

<table>
<thead>
<tr>
<th>Month</th>
<th>OLR(clear)</th>
<th>ULW</th>
<th>G(clear)</th>
<th>g(clear)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 2019</td>
<td>263.904</td>
<td>391.744</td>
<td>127.84</td>
<td>0.3263</td>
</tr>
<tr>
<td>Jan 2020</td>
<td>263.279</td>
<td>390.911</td>
<td>127.632</td>
<td>0.3265</td>
</tr>
<tr>
<td>Feb 2020</td>
<td>264.479</td>
<td>392.926</td>
<td>128.447</td>
<td>0.3269</td>
</tr>
<tr>
<td>Mar 2020</td>
<td>264.877</td>
<td>396.008</td>
<td>131.131</td>
<td>0.3311</td>
</tr>
<tr>
<td>Apr 2020</td>
<td>265.558</td>
<td>400.41</td>
<td>134.852</td>
<td>0.3368</td>
</tr>
<tr>
<td>May 2020</td>
<td>267.679</td>
<td>404.095</td>
<td>136.416</td>
<td>0.3376</td>
</tr>
<tr>
<td>Jun 2020</td>
<td>269.113</td>
<td>407.299</td>
<td>138.186</td>
<td>0.3393</td>
</tr>
<tr>
<td>Jul 2020</td>
<td>270.143</td>
<td>408.587</td>
<td>138.444</td>
<td>0.3388</td>
</tr>
<tr>
<td>Aug 2020</td>
<td>270.527</td>
<td>407.728</td>
<td>137.201</td>
<td>0.3365</td>
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<tr>
<td>Sep 2020</td>
<td>268.54</td>
<td>405.008</td>
<td>136.468</td>
<td>0.3370</td>
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<tr>
<td>Oct 2020</td>
<td>266.475</td>
<td>399.627</td>
<td>133.152</td>
<td>0.3332</td>
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<tr>
<td>Nov 2020</td>
<td>264.73</td>
<td>394.438</td>
<td>129.709</td>
<td>0.3288</td>
</tr>
<tr>
<td><strong>Data</strong></td>
<td><strong>266.61</strong></td>
<td><strong>399.90</strong></td>
<td><strong>133.29</strong></td>
<td><strong>0.3333</strong></td>
</tr>
<tr>
<td><strong>Theory</strong></td>
<td><strong>266.80</strong></td>
<td><strong>400.20</strong></td>
<td><strong>133.40</strong></td>
<td><strong>0.3333</strong></td>
</tr>
</tbody>
</table>

**THEORY:**

$g(\text{clear})$

Eq. (1) $B_g - B_0 = B_{\text{eff}}/2$

Eq. (3) $B_g = 2B_{\text{eff}}$

$B_G = B_0 - B_{\text{eff}}$ =>

$B_G : B_{\text{eff}} : B_0 : B_g = 5 : 10 : 15 : 20$

$G : OLR : ULW = 5 : 10 : 15$

$1 : 2 : 3 = 1 : 2 : 3$ =>

$g = G/ULW = 1/3$

$1 = 26.68 \text{ Wm}^{-2}$
### DATA: TOA SW up is integer

**CERES EBAF Ed4.1, Dec 2000 – Nov 2020**

<table>
<thead>
<tr>
<th>TOA Flux (clear-sky with ΔC)</th>
<th>N</th>
<th>Data</th>
<th>Theory</th>
<th>Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>SW up clear-sky</em></td>
<td>8 / 4</td>
<td>53.72</td>
<td>53.36</td>
<td>0.36</td>
</tr>
<tr>
<td><em>SW up all-sky</em></td>
<td>15 / 4</td>
<td>98.97</td>
<td>100.05</td>
<td>-1.08</td>
</tr>
<tr>
<td>LW up clear-sky</td>
<td>40 / 4</td>
<td>266.04</td>
<td>266.80</td>
<td>-0.76</td>
</tr>
<tr>
<td>LW up all-sky</td>
<td>36 / 4</td>
<td>240.26</td>
<td>240.12</td>
<td>0.14</td>
</tr>
<tr>
<td>TOA SW CRE</td>
<td>-7 / 4</td>
<td>-45.25</td>
<td>-46.69</td>
<td>1.44</td>
</tr>
<tr>
<td>TOA LW CRE</td>
<td>4 / 4</td>
<td>25.78</td>
<td>26.68</td>
<td>-0.90</td>
</tr>
<tr>
<td>TOA Net CRE</td>
<td>-3 / 4</td>
<td>-19.47</td>
<td>-20.01</td>
<td>0.54</td>
</tr>
<tr>
<td>Albedo, clear</td>
<td>8 / 51</td>
<td>0.158</td>
<td>0.157</td>
<td>0.001</td>
</tr>
<tr>
<td>Albedo, all</td>
<td>15 / 51</td>
<td>0.291</td>
<td>0.294</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

Clear-sky:  SW up =  8  SW in =  43  LW up = 40  TOA Net IMB =  3

All-sky:     SW up = 15  SW in = 36  LW up = 36  TOA Net CRE = -3

With TSI = 51, each flux is an **integer** on the intercepting cross-section disk.
**Data: TSI**

$$\text{TSI} = 51 \Rightarrow 1 = \text{LWCRE (spherical weighting)}$$

$$\text{TSI} = 51 \Rightarrow \text{UNIT} = (\text{TSI}/51) \times 4/4.0034 \quad \text{(geodetic weighting)}$$

- **TSI = 1361.03**

  $$1 = \text{LWCRE} = 26.69 \quad \text{(spherical weighting)}$$

  $$1 = \text{UNIT} = 26.67 \quad \text{(geodetic weighting)}$$

---

**Total Solar Irradiance for CERES Edition-4 20000301-20210131**

- **TSI Mean = 1361.03 W m}^{-2}**

**Normalized SORCE V15 (from V15 - V19)**

- WRC Composite [01MAR2000 - 24FEB2003]
- RMIB Composite [01JUL2013 - 31OCT2014]
- SORCE/TsI-1 Composite [01JAN2018 - 25FEB2020]
- TSIS-1 V3 [26FEB2020 - 31JAN2021]

*Normalized to SORCE V15*
(1361.63/51) × (4/4.0034) = 1 = 26.68 ± 0.01 Wm⁻²
The Greenhouse Effect of Clouds: \( \Delta F(\text{CERES}) = -0.09 \) Wm\(^{-2}\)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLR all-sky</td>
<td>240.26</td>
</tr>
<tr>
<td>LWCRE CERES Mean</td>
<td>26.59</td>
</tr>
<tr>
<td>OLR clear-sky</td>
<td>266.04</td>
</tr>
<tr>
<td>LWCRE Theory</td>
<td>26.68</td>
</tr>
<tr>
<td>LW net clear</td>
<td>81.02</td>
</tr>
<tr>
<td>LWCRE at surface</td>
<td>27.40</td>
</tr>
<tr>
<td>LW net all-sky</td>
<td>53.62</td>
</tr>
</tbody>
</table>

Stephens et al. (2012)

LWCRE mean = 26.65 Wm\(^{-2}\)

\( \Delta F(\text{Stephens}) = -0.03 \) Wm\(^{-2}\)

LWCRE Theory

\[ TSI = 51 \]

4 Eqs 20-yr

\[ 1 = 26.68 \text{ Wm}^{-2} \]
The Shielding Effect of Clouds: $\Delta F(\text{CERES}) = 1.44 \ \text{Wm}^{-2}$

<table>
<thead>
<tr>
<th>RSR all-sky CERES</th>
<th>SWCRE at TOA</th>
<th>RSR clear-sky</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.97</td>
<td>$-45.25$</td>
<td>53.72</td>
</tr>
</tbody>
</table>

N-position

15/4 = 100.05

$-7/4 = -46.69$

8/4 = 53.36

\[
\text{TOA Net CRE} = \text{LWCRE} + \text{SWCRE} =
\]

\[
= \text{OLR(clear)} - \text{OLR(all)} + \text{RSR(clear)} - \text{RSR(all)} =
\]

\[
= \text{OLR(clear)} - \text{OLR(all)} + \text{ISR} - \text{ASR(clear)} - [\text{ISR} - \text{ASR(all)}] =
\]

\[
= [\text{ASR(all)} - \text{OLR(all)}] - [(\text{ASR(clear)} - \text{OLR(clear)}] =
\]

\[
= \text{TOA Net IMB(all)} - \text{TOA Net IMB(clear)}
\]
The Shielding Effect of Clouds: $\Delta F(\text{CERES}) = 1.44 \text{ Wm}^{-2}$

<table>
<thead>
<tr>
<th>RSR all-sky CERES</th>
<th>SWCRE at TOA</th>
<th>RSR clear-sky</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.97</td>
<td>$-45.25$</td>
<td>53.72</td>
</tr>
</tbody>
</table>

$\text{N-position} \quad 15/4 = 100.05 \quad \text{SWCRE TOA N-position} \quad -7/4 = -46.69 \quad \text{N-position} \quad 8/4 = 53.36$

$\Rightarrow \text{EEI} = \text{TOA Net CRE} + \text{TOA Net IMB (clear)}$

**DATA**

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
<th>Net CRE</th>
<th>Net IMB (clear)</th>
<th>Net IMB (all)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.78 – 45.25</td>
<td>TOA Net CRE</td>
<td>$-19.47 \text{ Wm}^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>340.01 – 53.72 – 266.04</td>
<td>TOA Net IMB (clear)</td>
<td>20.25 $\text{ Wm}^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EEI</td>
<td>TOA Net IMB (all)</td>
<td>0.78 $\text{ Wm}^{-2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**THEORY**

<table>
<thead>
<tr>
<th>Description</th>
<th>Net CRE</th>
<th>Net IMB (clear)</th>
<th>Net IMB (all)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOA Net CRE</td>
<td>$4/4 + 8/4 - 15/4 = -3/4 = -20.01 \text{ Wm}^{-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOA Net IMB (clear)</td>
<td>$51/4 - 8/4 - 40/4 = 3/4 = 20.01 \text{ Wm}^{-2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>