Sampling for wintertime surface conductive flux at the sea-ice/atmosphere interface

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CERES Science Team Meeting

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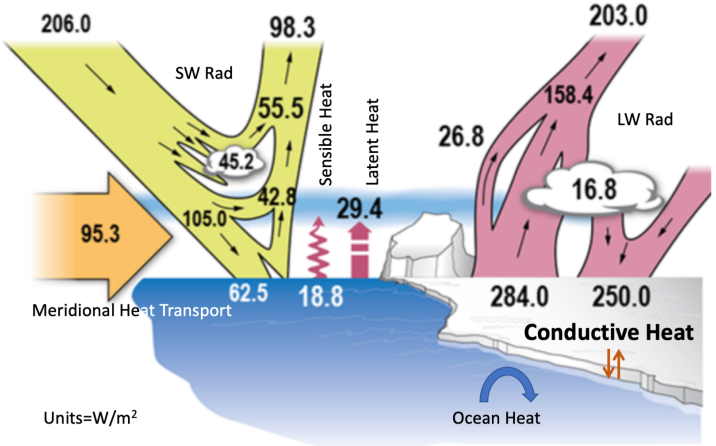
Motivation

Surface energy balance on wintertime sea-ice



Adapted from Kiehl & Trenberth, 1997, BAMS, by the JPL Graphic Arts Department

- *What is the expression of heat at the surface/atmosphere boundary?*
- *What is the spatial regime on either side of the boundary for which the surface energy balance equation is valid?*
- *Explicitly express spatio-temporal evolution of temperature profile in ice, given surface energy balance equation.*

CICE consortium discretization

	Lower boundary values				
distribution	**original**	**round**	**WMO**		
`kcatbound`	0	1	2		
N_C	5	5	5	6	7
categories	lower bound (m)				
1	0.00	0.00	0.00	0.00	0.00
2	0.64	0.60	0.30	0.15	0.10
3	1.39	1.40	0.70	0.30	0.15
4	2.47	2.40	1.20	0.70	0.30
5	4.57	3.60	2.00	1.20	0.70
6				2.00	1.20
7					2.00

https://cice-consortium-icepack.readthedocs.io/en/master/science_guide/sg_itd.html#tab-itd

Sea Ice model

Surface Cooling

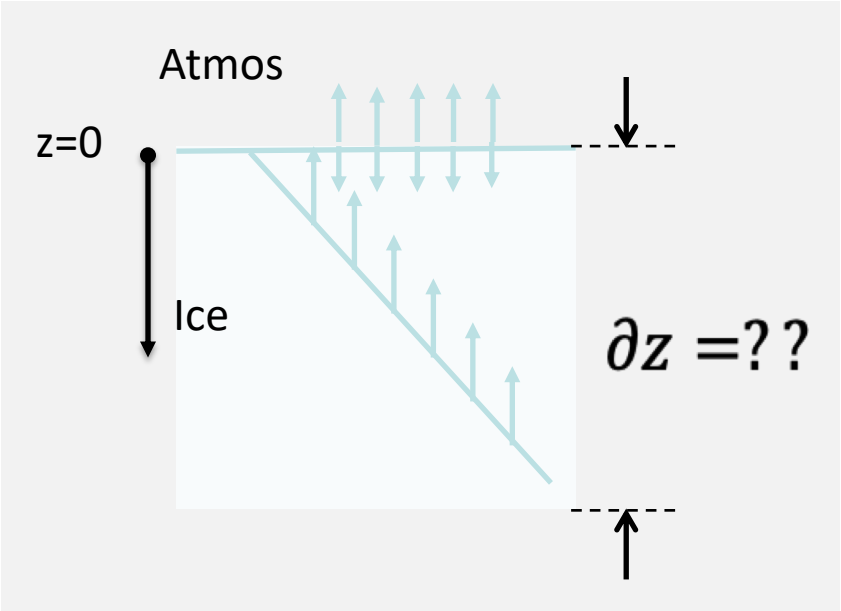
$$K \left.\frac{\Delta T(z,y)}{\Delta z}\right|_{z=0} > \sigma[\varepsilon_s T_s^4 - \varepsilon_{atm} T_{atm}^4] + c_s[\theta_{air} - T_s(t)]$$

Surface melt

$$K \left.\frac{\Delta T(z,y)}{\Delta z}\right|_{z=0} < \sigma[\varepsilon_s T_s^4 - \varepsilon_{atm} T_{atm}^4] + c_s[\theta_{air} - T_s(t)]$$

Wintertime Surface Energy Balance

$$K \left.\frac{\partial T(z,y)}{\partial z}\right|_{z=0} = \sigma[\varepsilon_s T_s^4 - \varepsilon_{atm} T_{atm}^4] + c_s[\theta_{air} - T_s(t)]$$



What is the physical regime for ∂z?

Heat diffusion in ice

$$\frac{\partial^2 T}{\partial z^2} - \frac{1}{\kappa}\frac{\partial T}{\partial t} = 0$$

Wintertime Surface Energy Balance

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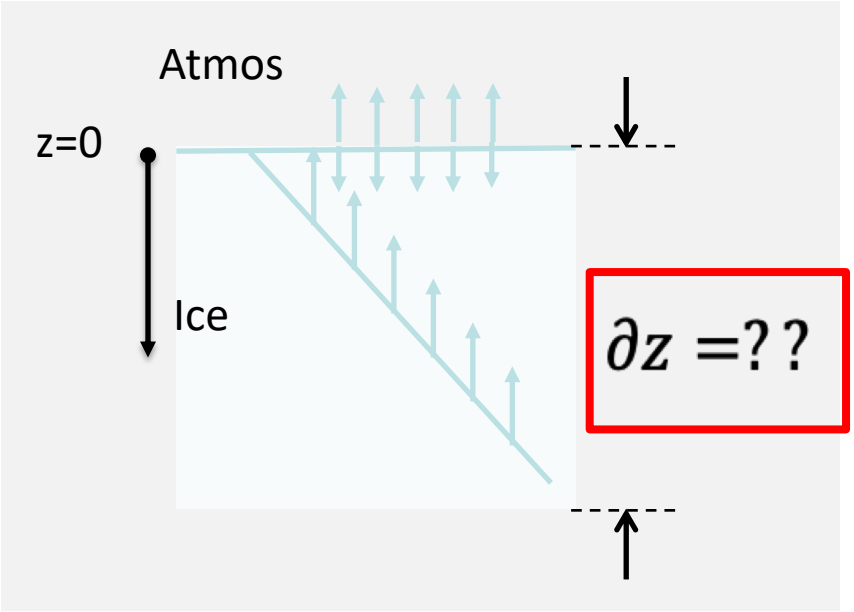


$\partial z = ??$

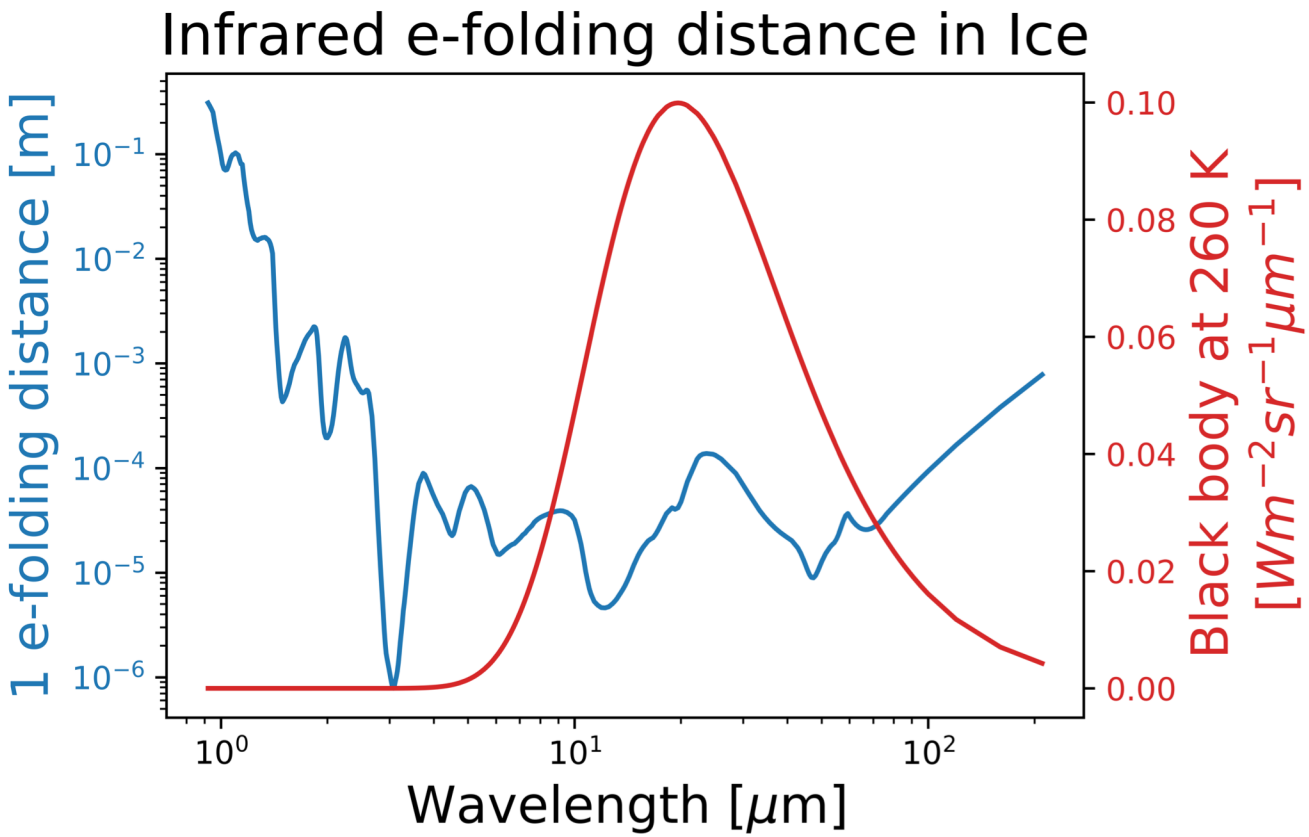
What is the physical regime for ∂z?

Heat diffusion in ice

$$\frac{\partial^2 T}{\partial z^2} - \frac{1}{\kappa}\frac{\partial T}{\partial t} = 0$$

LW cooling to atmosphere ($\varepsilon\sigma T_s^4$) is limited to sub-mm e-folding distances in ice.

∂z < 1 mm



Heat diffusion equation solved using Green's function approach

For an initial temperature distribution = $\sum_{\nu=0}^{N} g_\nu z^\nu$, ,a polynomial in z of order N and coefficients g_ν .

<u>Temperature distribution in ice</u>

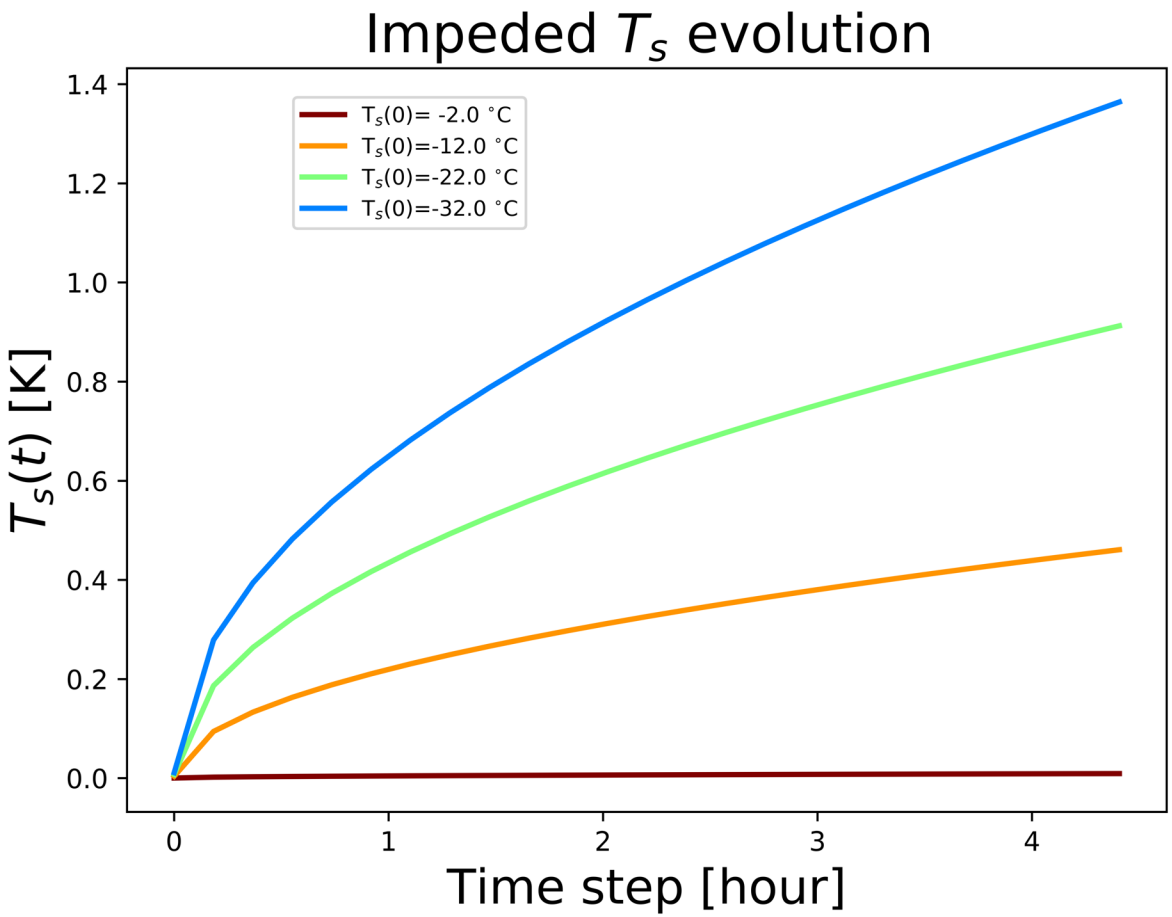
$$
\begin{aligned}
T(z,t) \quad = \quad & \frac{1}{2\sqrt{\pi\kappa t}} \int_0^\infty \sum_{\nu=0}^{N} g_\nu z'^\nu \left[e^{-(z-z')^2/4\kappa t} - e^{-(z+z')^2/4\kappa t} \right] dz' \\
& + \sum_{\nu=0}^{N} g_\nu \frac{\Gamma[(\nu+1)/2]}{\sqrt{\pi}} (4\kappa)^{\nu/2} \\
& \times \sum_{n=0}^{\infty} a_n \frac{\Gamma[(n+\nu)/2+1]}{n!} (4t)^{(n+\nu)/2} i^{n+\nu} \mathrm{erfc}\left[\frac{z}{2\sqrt{\kappa t}} \right]
\end{aligned}
$$

<u>Coefficient a_0 is the impeded heat</u>

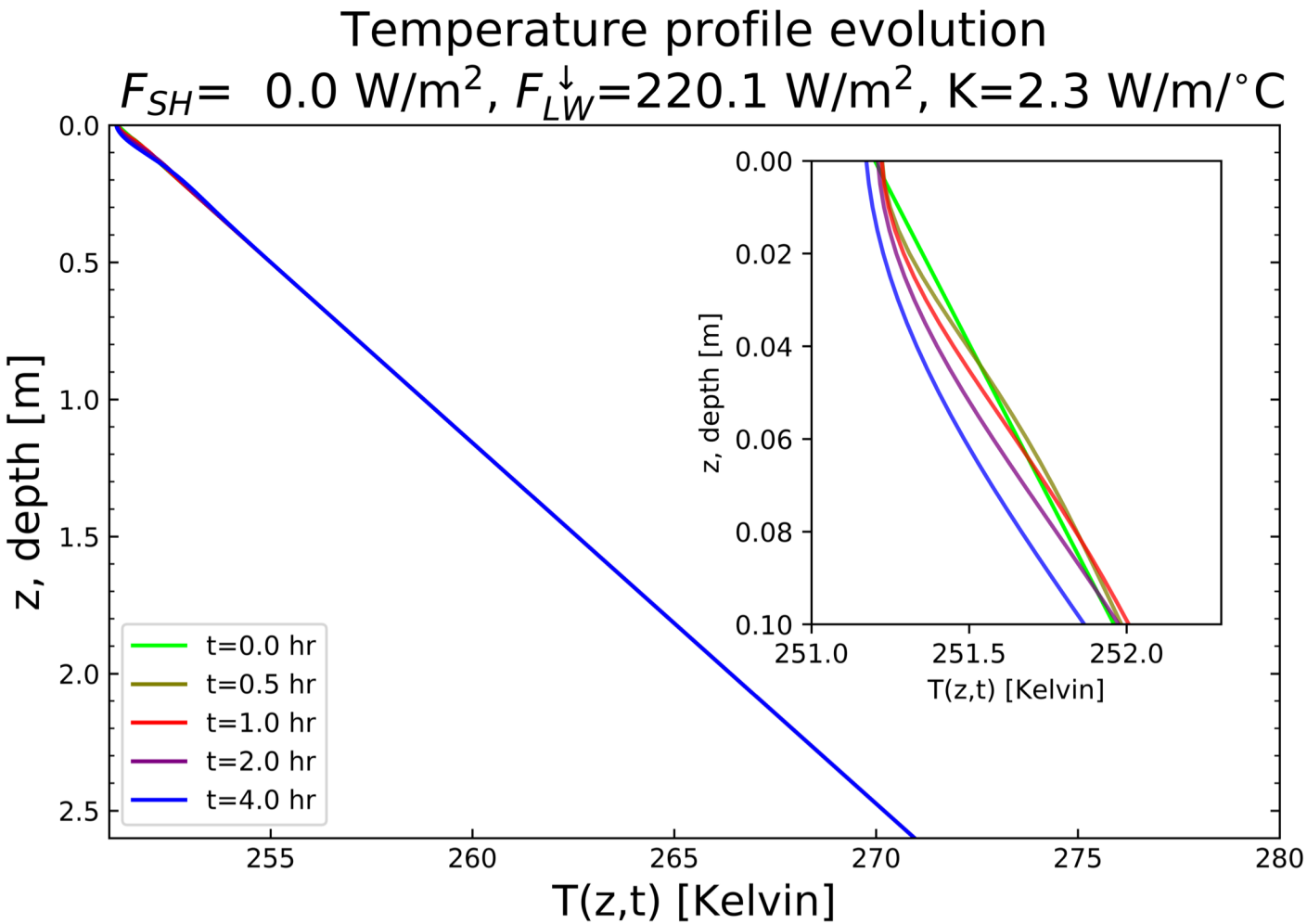
$$
\begin{aligned}
a_n \quad = \quad & \left[-\frac{n!\sqrt{\pi}}{K g_0} \frac{\Gamma[n/2+1/2]}{\Gamma[n/2+1]} \right] \times \\
& \left[\varepsilon\sigma \sum_{h}^{N_g} \zeta_h \frac{b_{n-N_g+h}}{!n-N_g+h)!} + \sum_{i=0}^{N} \frac{a_{n-i-1}}{(n-i-1)!} \frac{c_s}{\sqrt{\pi}} g_i \Gamma[(i+1)/2](4\kappa)^{i/2} \right. \\
& \left. + \quad K \sum_{j=1}^{N} \frac{a_{n-j}}{(n-j)!} g_j \frac{\Gamma[(j+1)/2]}{\sqrt{\pi}} \frac{\Gamma[(n+j)/2+1]}{\Gamma[(n+j)/2+1/2]} (4\kappa)^{j/2} \right]
\end{aligned}
$$

κ is diffusivity, K is conductivity, z is position, t is time, T is temperature, T_s is surface temperature, T_{atm} is atmospheric temperature, θ is potential temperature, ε is snow/ice emissivity, σ is the Stefan Boltzmann constant, i, j, h, n and ν are indices, a_n are coefficients to be solved, c_s is the sensible heat flux coefficient, ς is a permutation coefficient, and N_g is a permutation order.

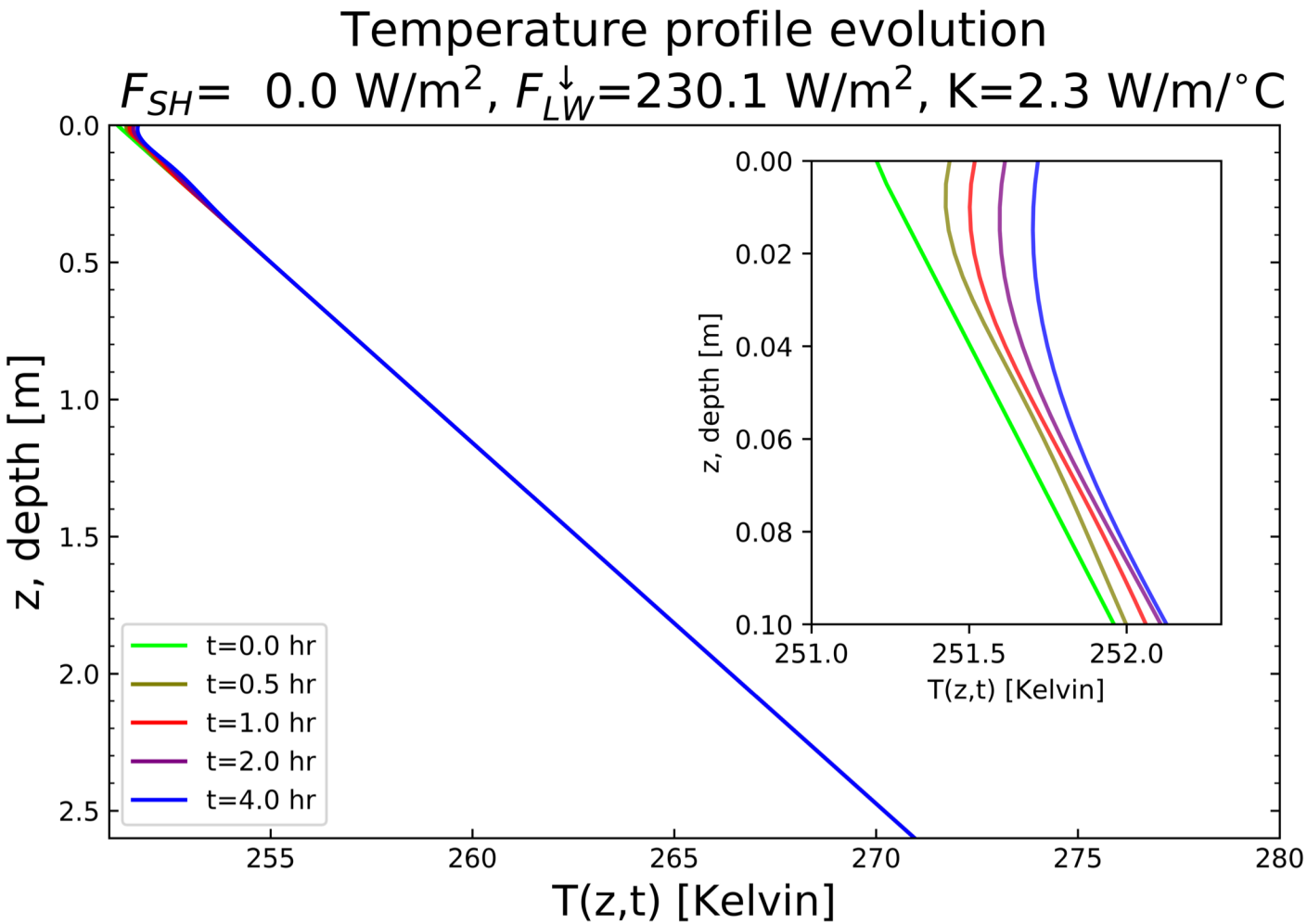
Impeded conducted heat as temperature



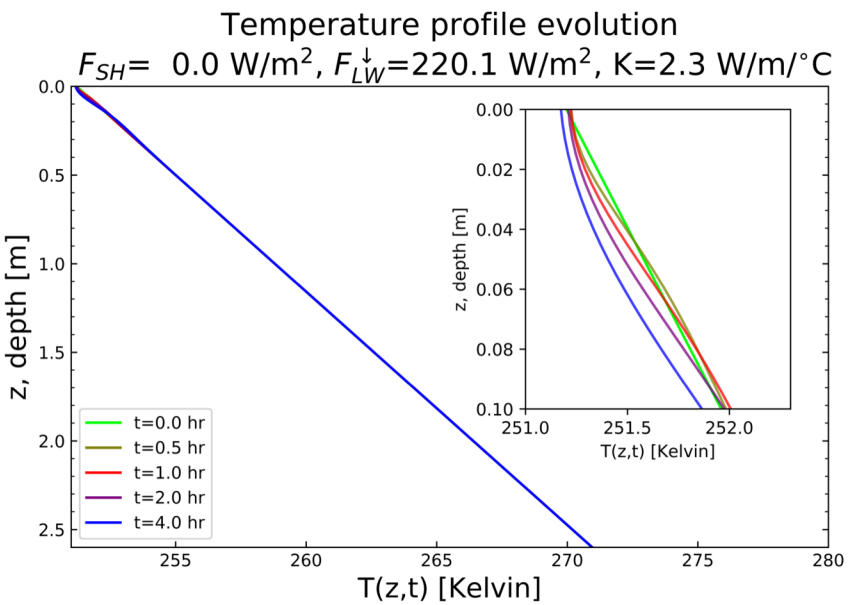
Solution stability test: Wintertime surface at equilibrium

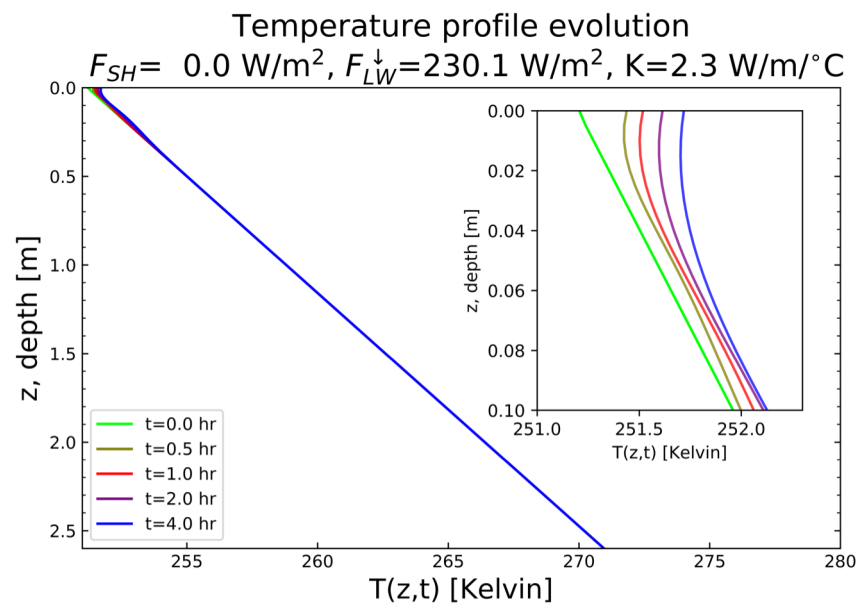


Wintertime surface forced from atmosphere



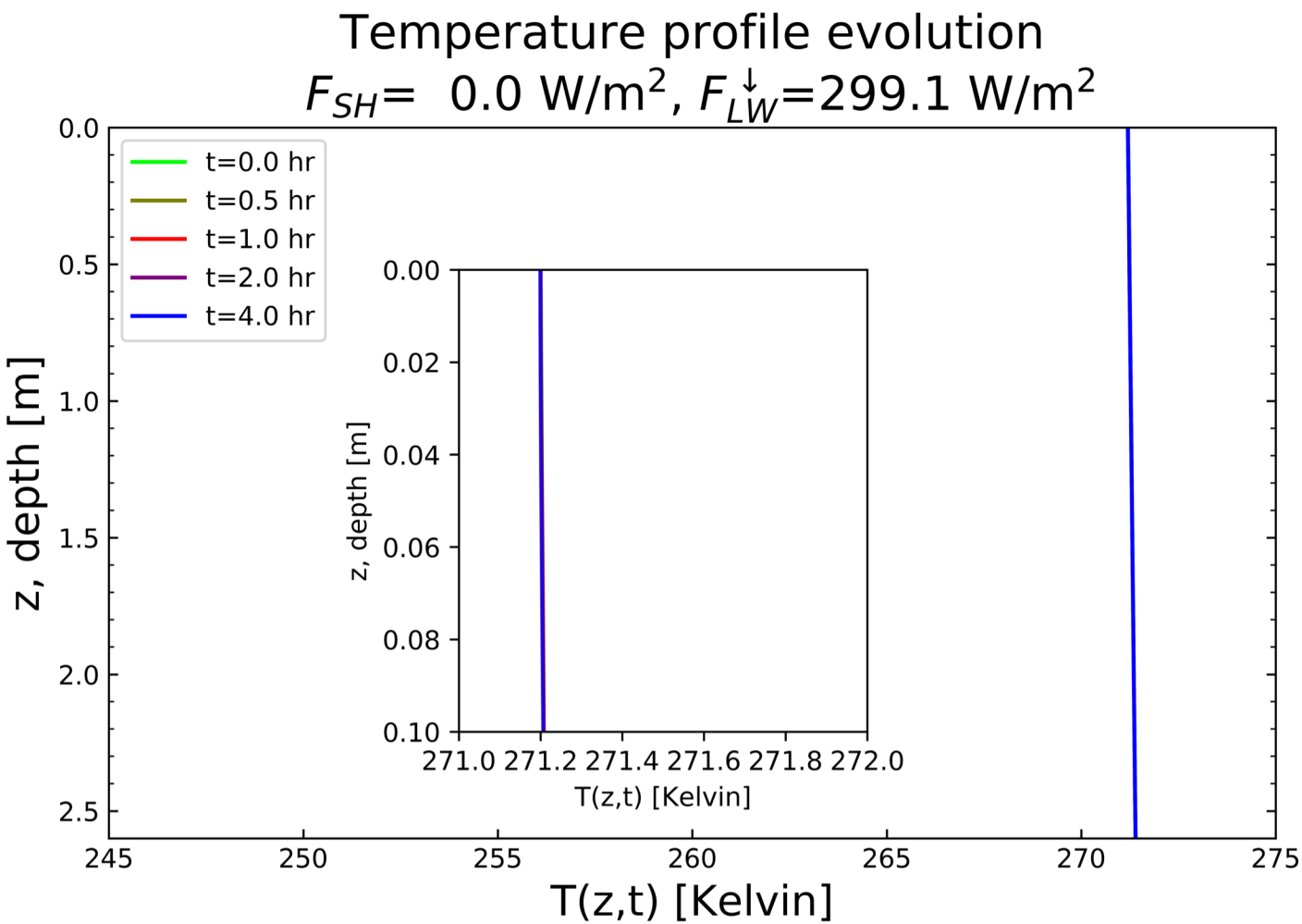
Wintertime Heat flux errors can > 10 W/m^2 If discretizing 1st layer at 10 cm



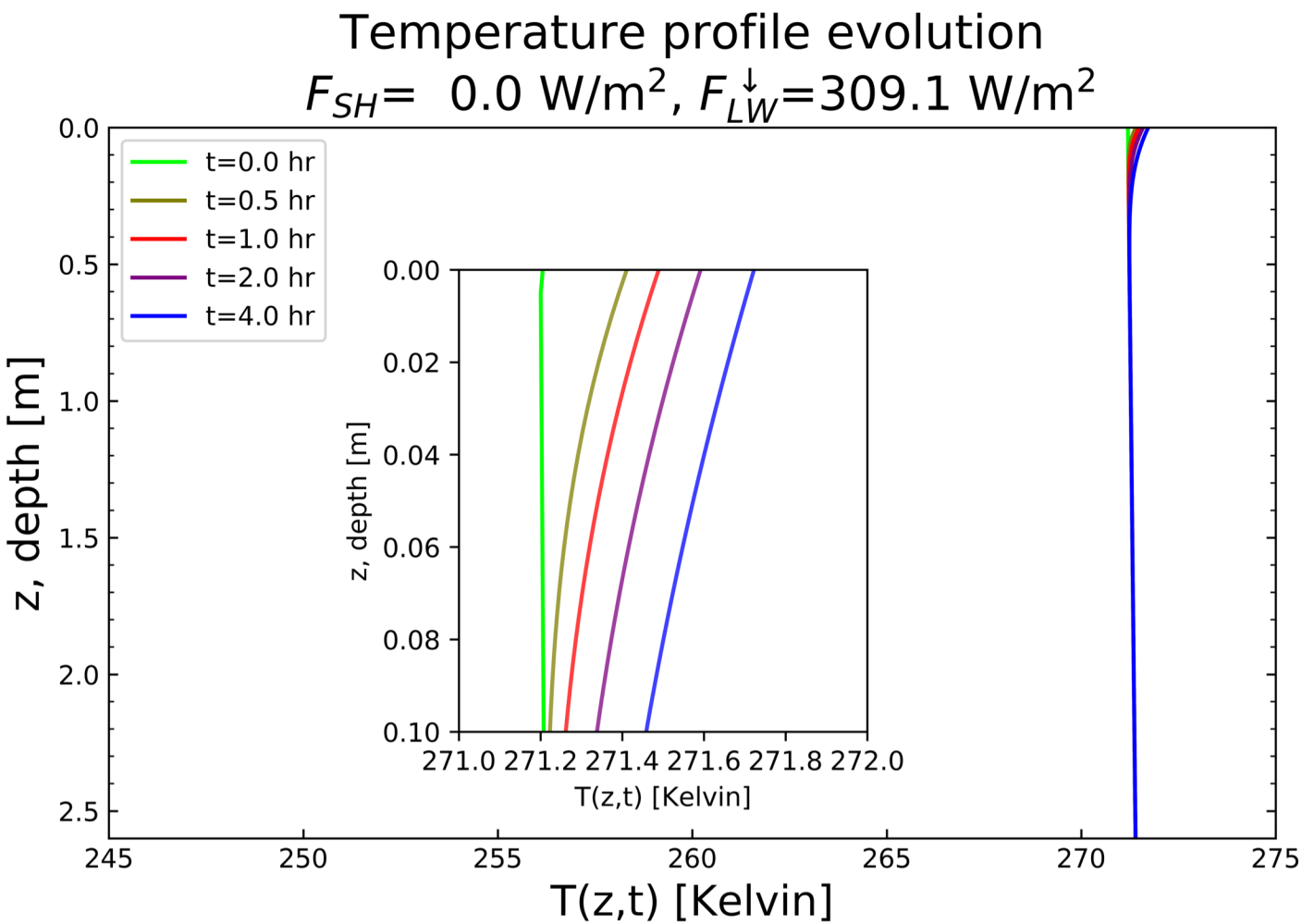


Discretization at 10 cm layer doesn't capture the curvature in wintertime temperature profile due to impedance

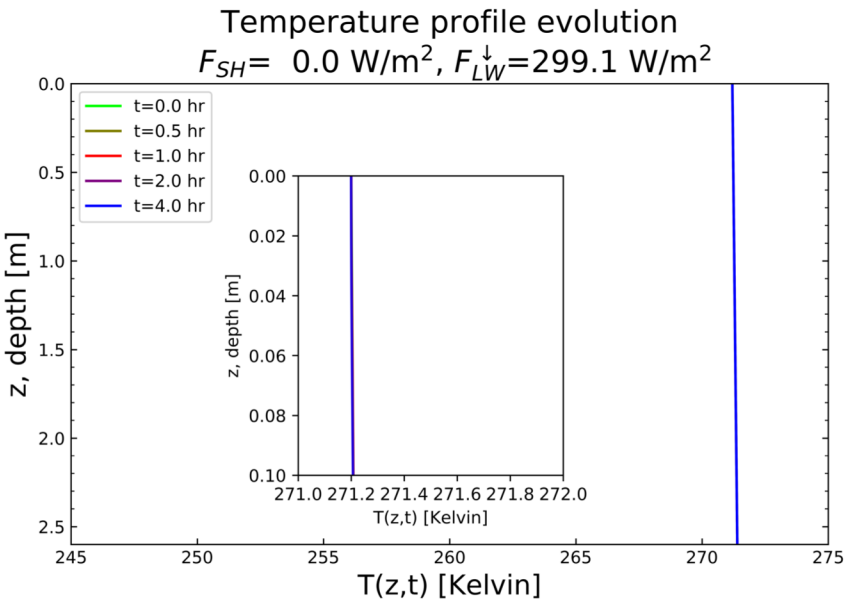
Solution stability test: Summertime surface at equilibrium

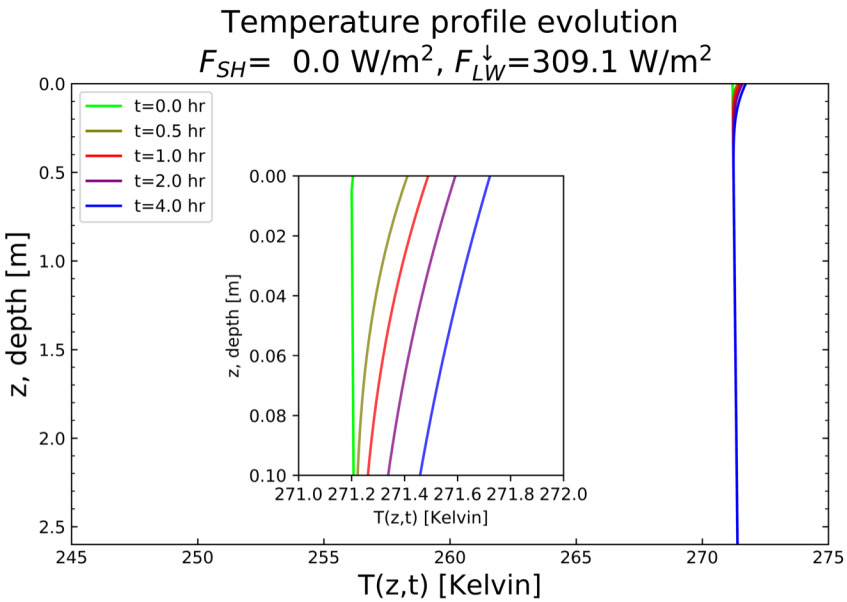


Summertime surface forced from atmosphere



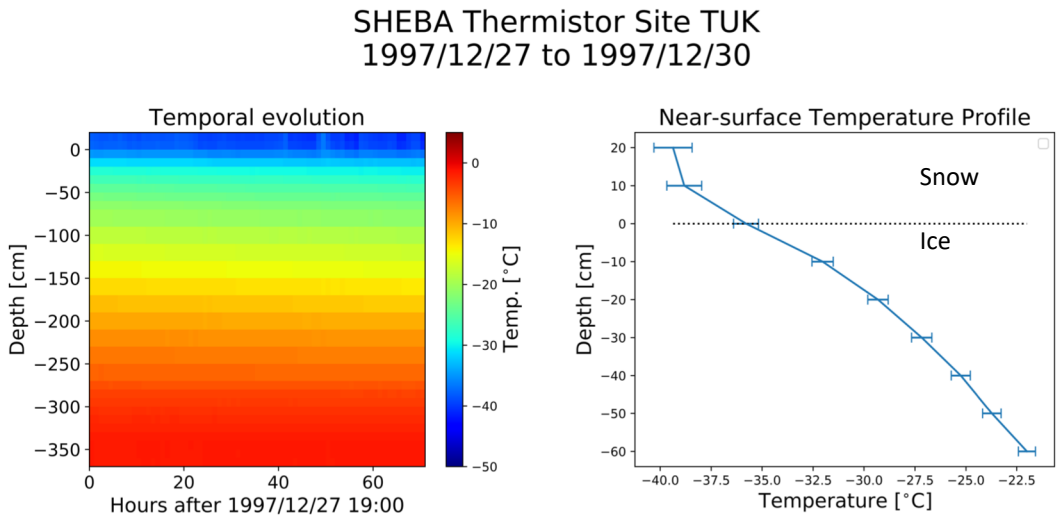
Summertime Heat flux errors negligible If discretizing 1st layer at 10 cm



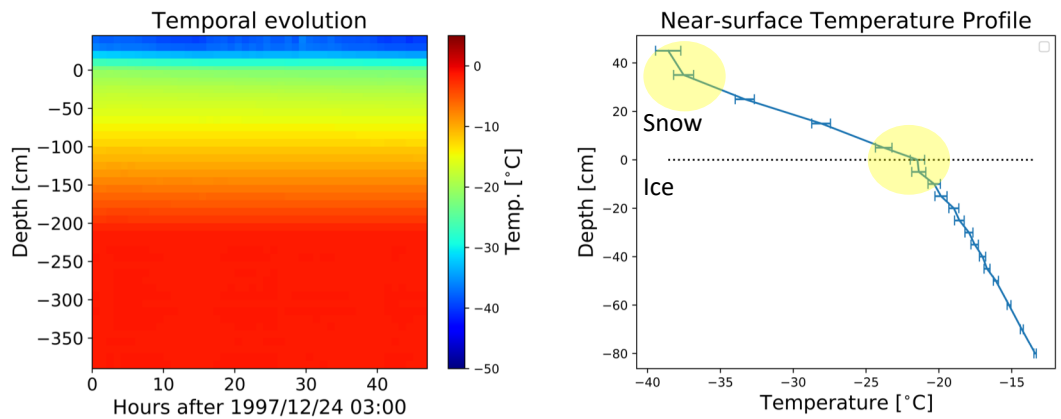


SHEBA Thermistor String: fine-spacing captures impedance

Coarse: 10-cm spacing



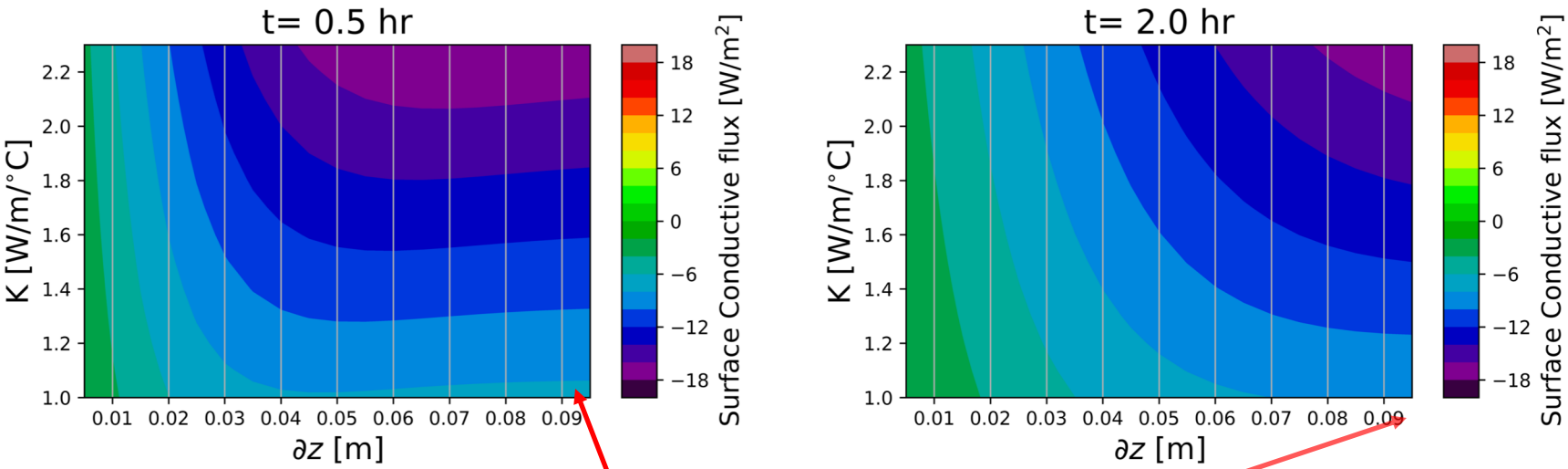
Fine: 5 cm-spacing



Numerical model ice layer thickness $\partial z \geq 10$ cm can result in wintertime surface conductive flux errors>10 W/m^2

Surface Conductive Flux dependence on ∂z, $-K\frac{\partial T(z,t)}{\partial z}$

$T_s(0)$=-22.0 °C, F_{SH}= 0.0 W/m^2, $F_{LW}^{\downarrow}$=220.1 W/m^2



CICE min. surface
$\partial z = 10$ cm

Summary…

- **Surface Energy Balance regime**
- Physical regime for energy flux at the boundary must be considered for correct energy balance.
- Atmospheric LW radiation penetrates only sub-mm in ice.
- Conductive heat flux layer thickness representation must approach zero in numerical models. $\partial z \rightarrow 0$ cm.
- The analytical derivation explicitly separates the instantaneous conducted heat flow available for cooling to the atmosphere and impeded heat.

…Summary

- **Wintertime**
- Conductive heat flows to the surface, and conductive heat is impeded at the ice/air interface.

- **Summertime**
- Same thickness requirements for . $\partial z \rightarrow 0$ cm, but current numerical implementation is inconsequential due to zero to downward conductive heat flow.

- Heat impedance may be observed in SHEBA temperature profiles. Temperature data from MOSAiC Conductivity Temperature Depth (CTD) profilers with thermistors spaced 2.5 cm apart can help validate the theoretical impedance.

Conclusions

•An analytical expression for temperature profile shows that the spatial regime of physical processes at the sea-ice/atmosphere boundary should only consider high resolution (~ < 1 cm) in numerical models.

•Due to impedance of upward heat flow, wintertime errors in surface energy balance can be >10 W/m2 for standardized sea-ice model layer thicknesses.