Observation-based decomposition of radiative perturbations and radiative kernels

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Seiji Kato
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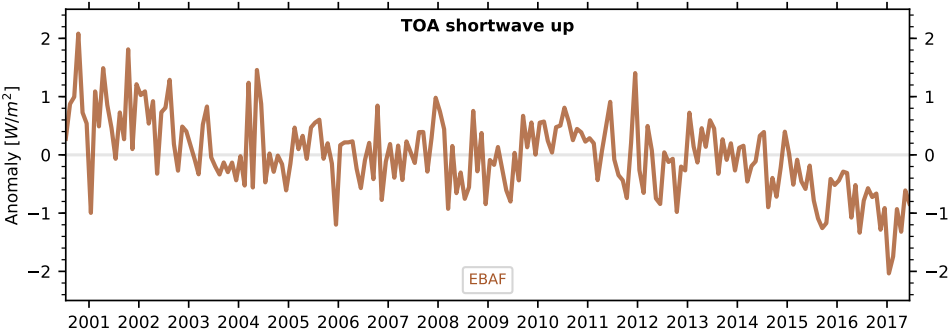
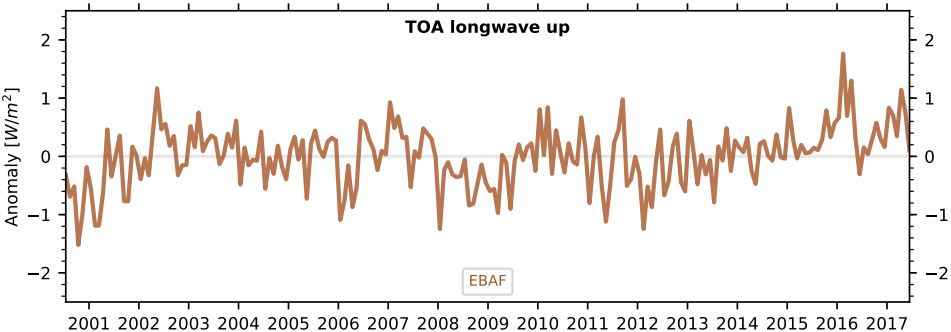
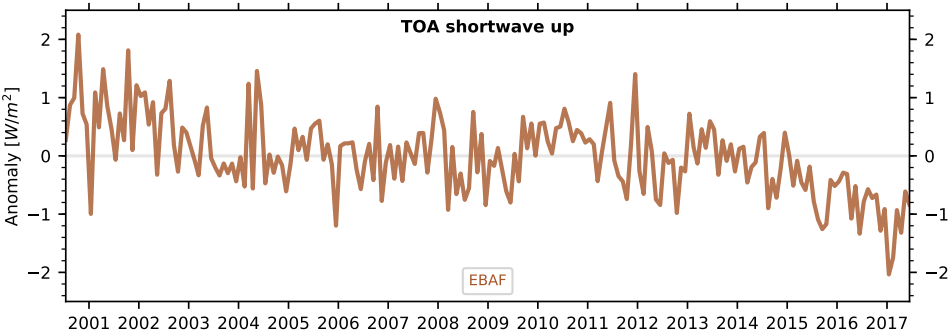
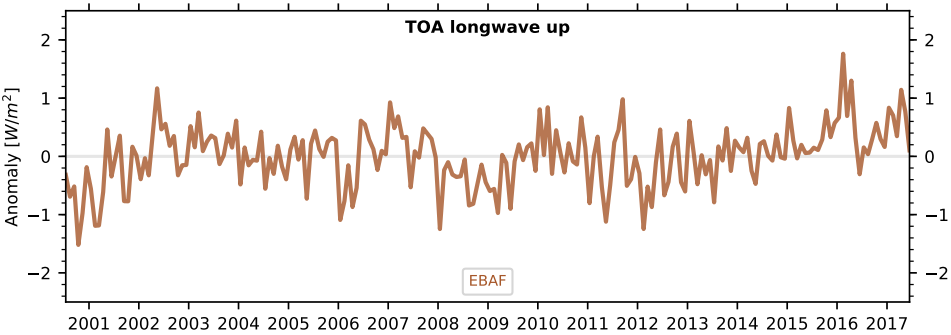


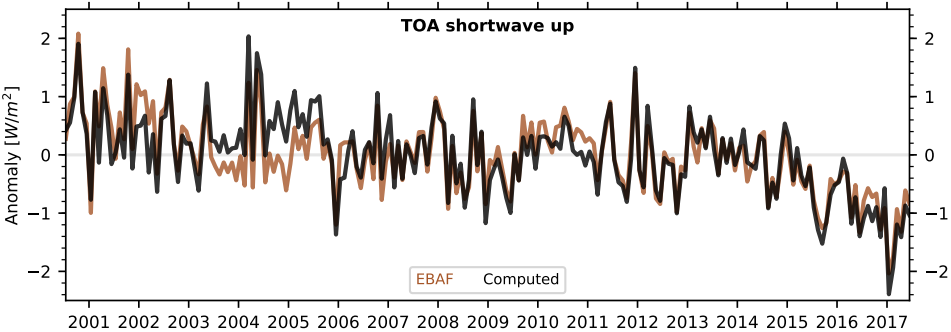


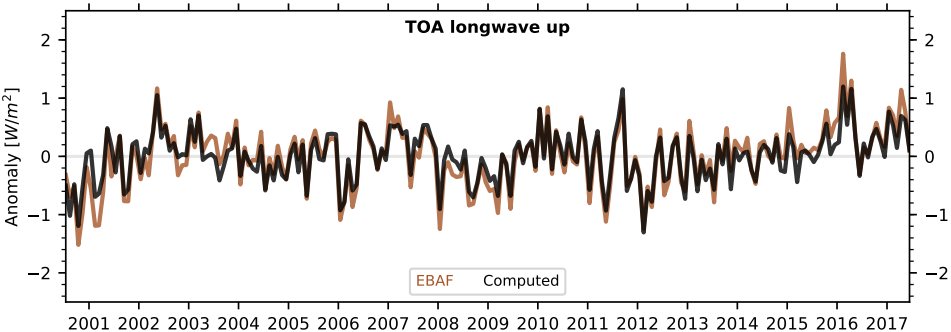
- CERES EBAF-TOA global mean anomalies: decline the SW; recent El-Nino in the LW
- What are the contributions from individual variables to these total flux anomalies?





- Can we compute fluxes that reasonably reproduce the EBAF variability?





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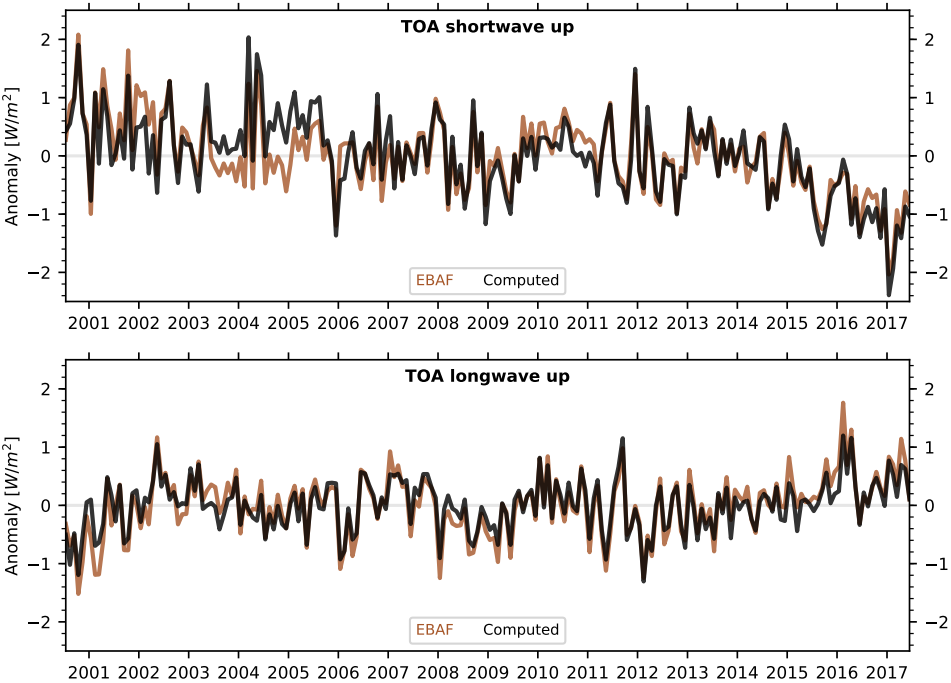
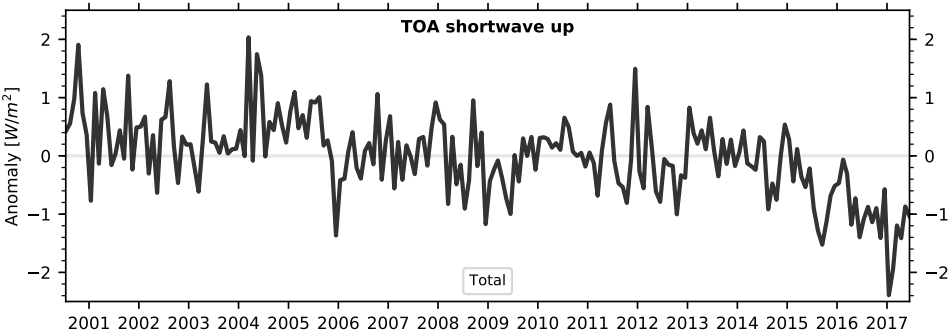
- Yes. Fluxes computed via radiative transfer and observed inputs (**Black**)



- Do calculations to decomposing the **total** anomalies...


TOA shortwave up
Anomaly [W/m²]
2
1
0
−1
−2
Total
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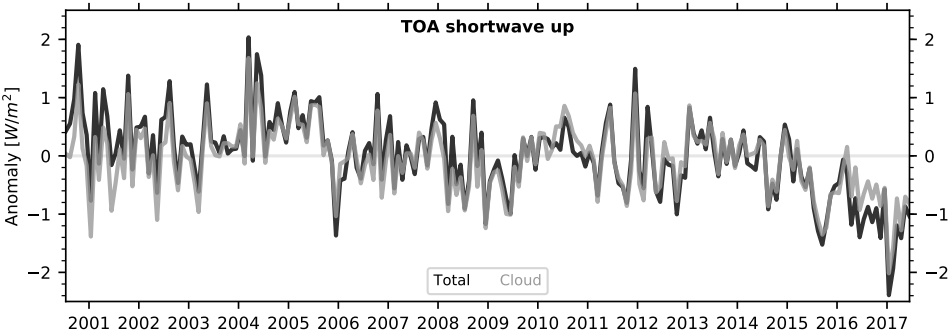

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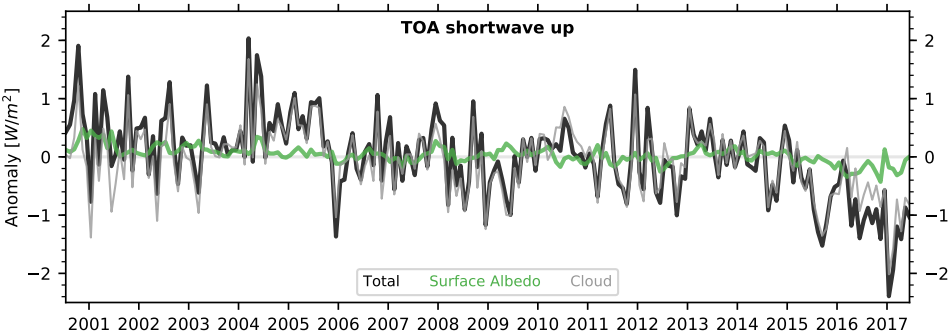

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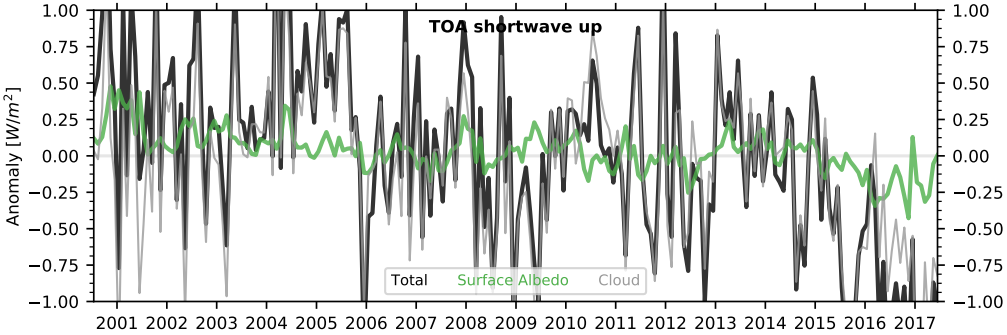

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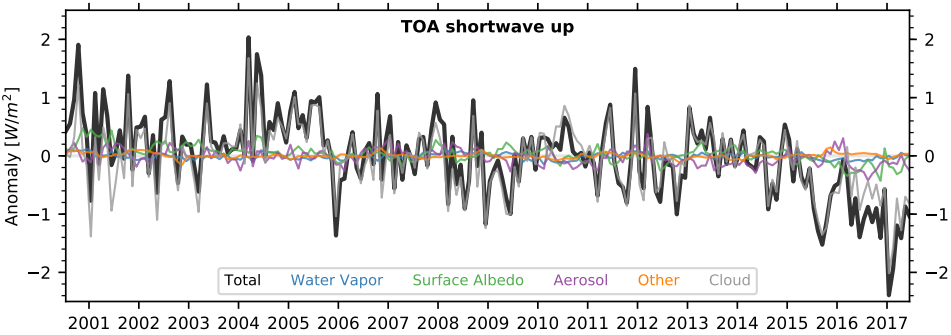

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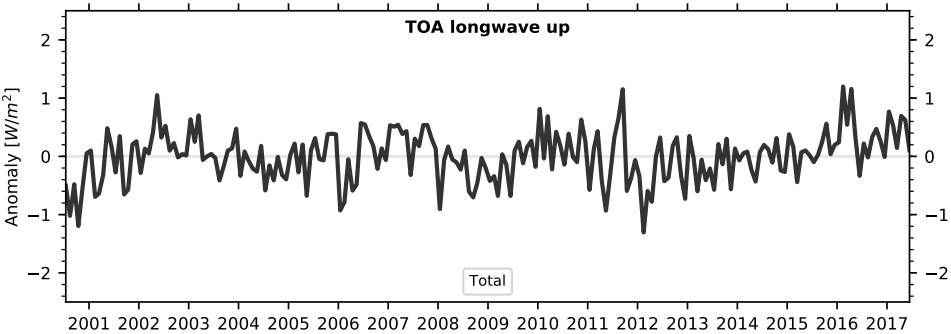
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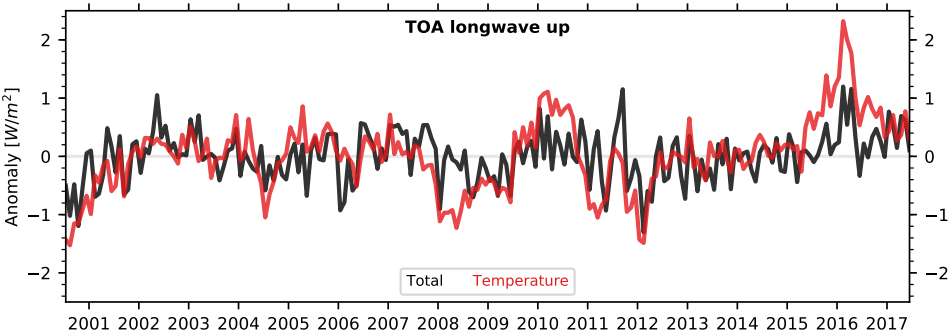

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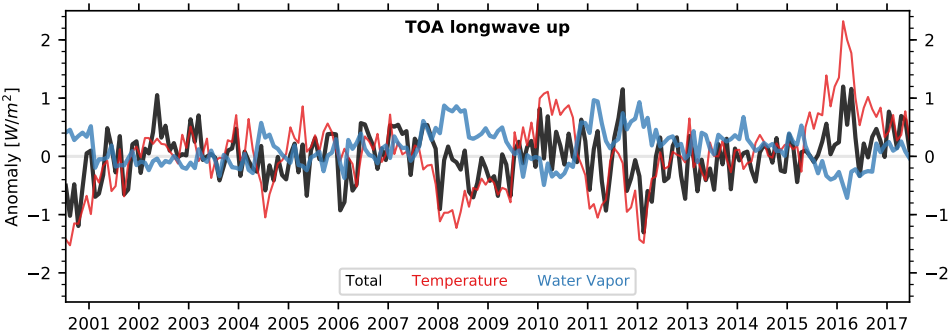

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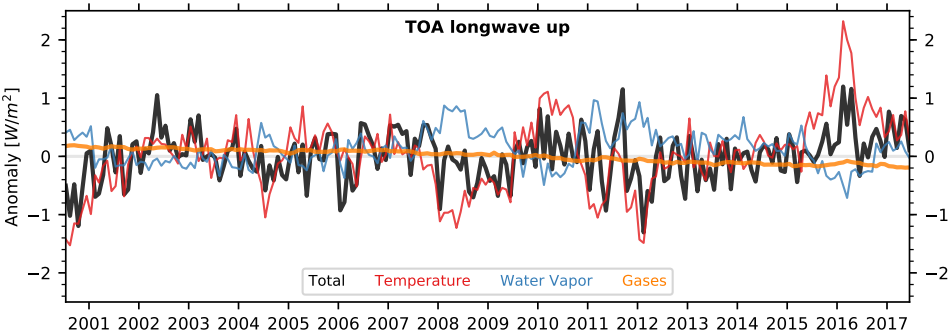
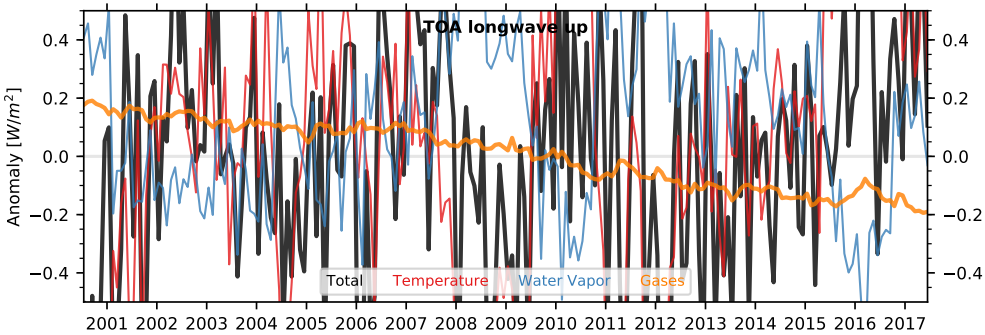
- Water vapor: anti-correlated with temperature



- Steady, gentle decrease in from trace gases (carbon dioxide)



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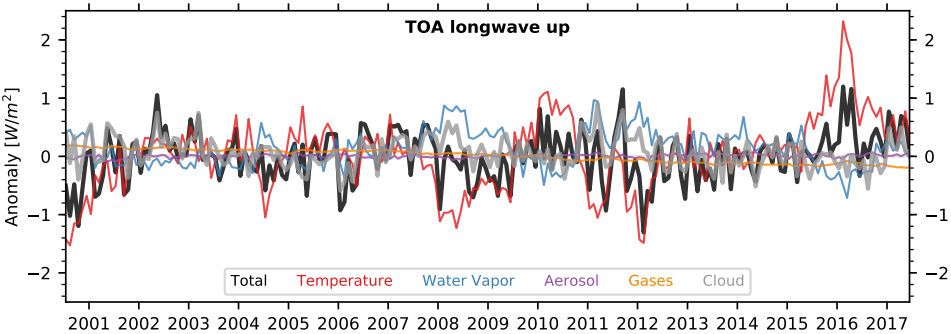
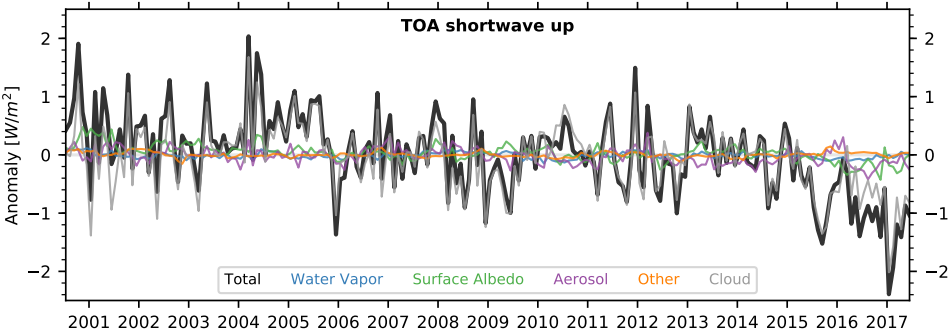
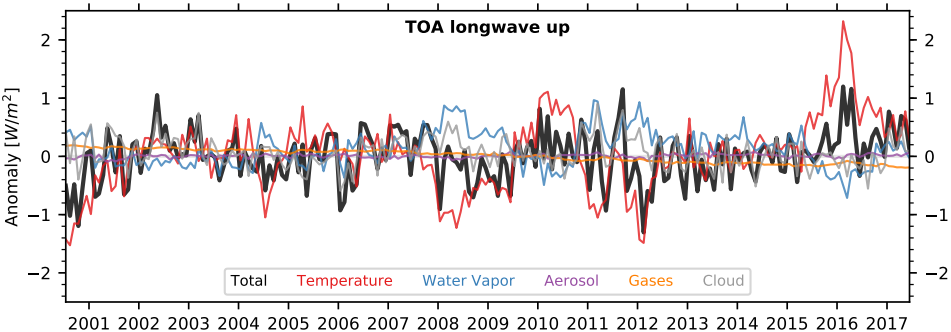


- Tiny effect of Aerosols
- Cloud contributions more complicated than the shortwave





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   - Multiple kernel by a larger Δ(water vapor) $\rightarrow$ corresponding radiative effect
   - Far fewer calculations, easy to consistently apply across different climate models

CERES-PRP

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- Just straightforward finite differences, allows for more flexible perturbations
- Kernel requires flux perturbation $\partial F_{\Delta x}$ and the perturbation itself Δx be linearly related for their ratio to be valid

Radiative kernels

Radiative kernels are very convenient, and are computed using the observed anomaly Δx

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- General insight into the variability of the observed radiation budget
- Observation-based alternative / validation of GCM kernels
- GCM kernels applied to observations: attractive to purely use observations

Datasets

- Gridded monthly mean values $\rightarrow$ NASA Langley Fu-Liou Radiative Transfer Model

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temperature, water vapor, ozone, skin temperature, surface pressure	GEOS, AIRS
carbon dioxide, methane	AIRS
nitrous oxide, CFC11, CFC12, HCFC22	NOAA-ESRL
cloud fraction, base pressure, top pressure, phase, optical depth, ice effective diameter, liquid effective radius	SYN, C3M, SSF, SSFC3M
surface albedo	CERES-SAH
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- C3M: combined CERES CALIPSO CloudSat MODIS (C3M) product

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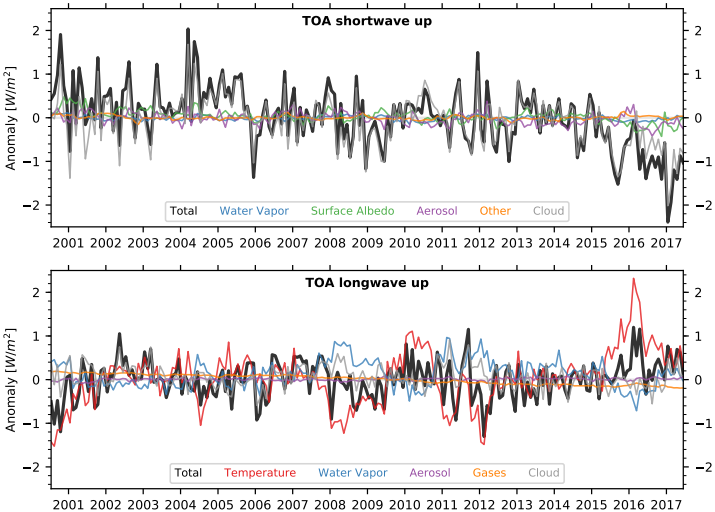

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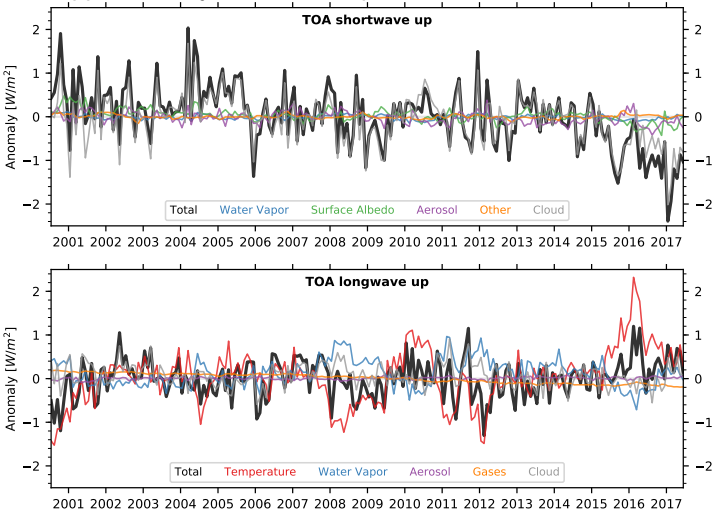

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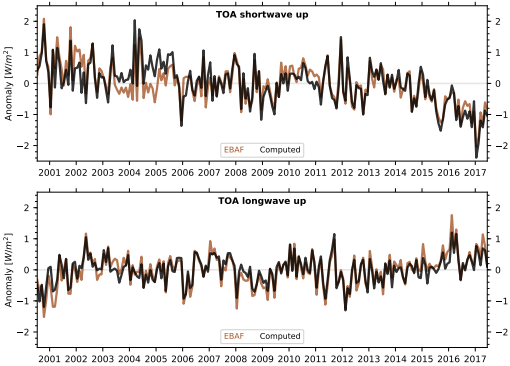

Comparison to EBAF fluxes

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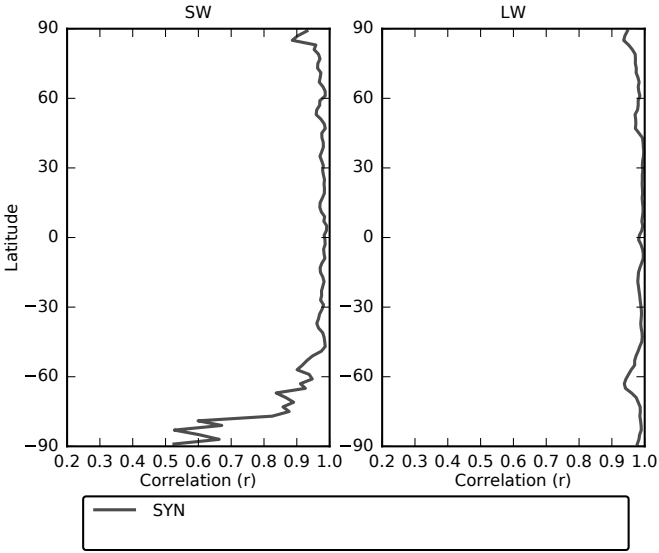

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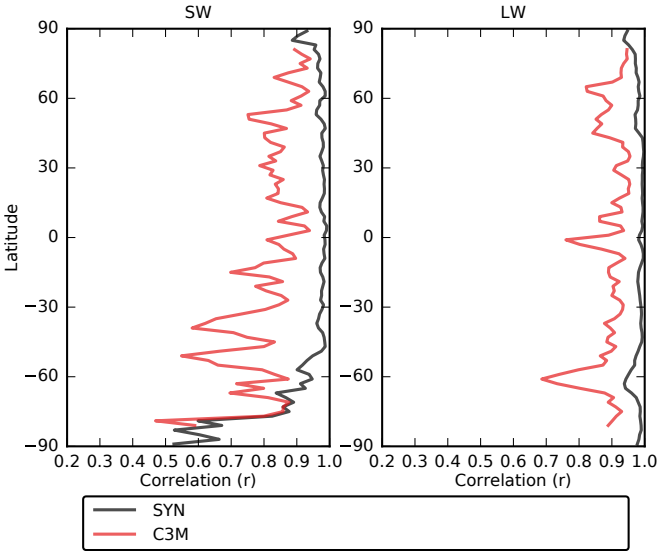

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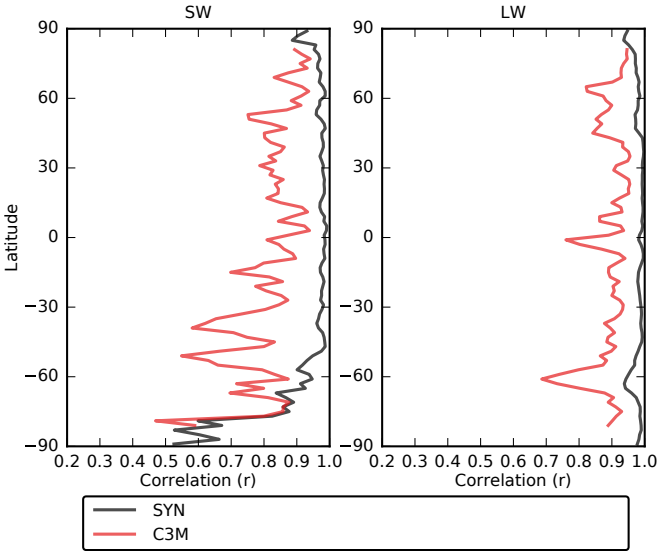

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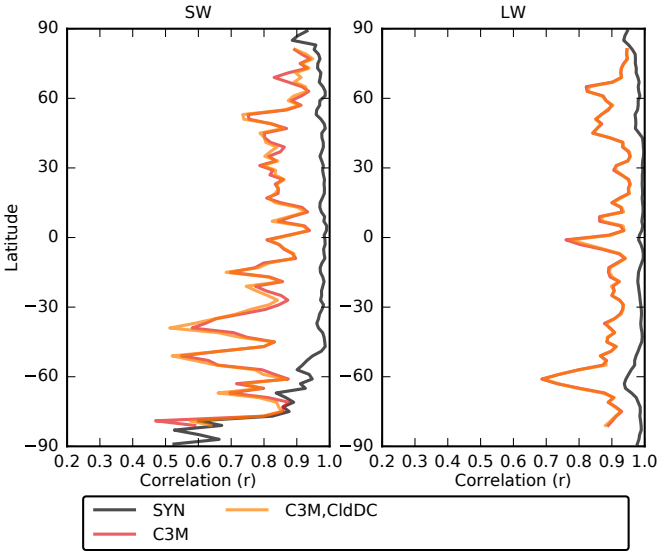

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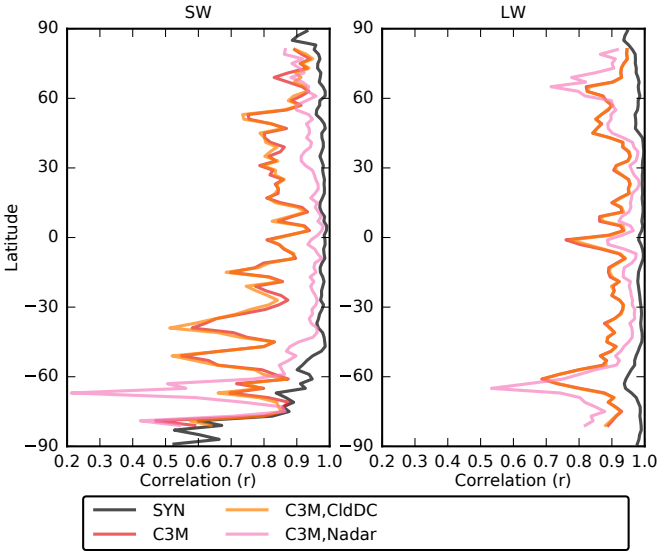

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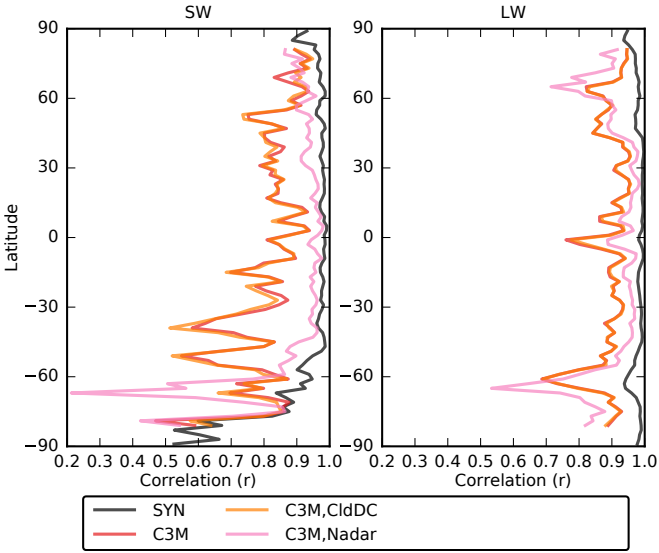
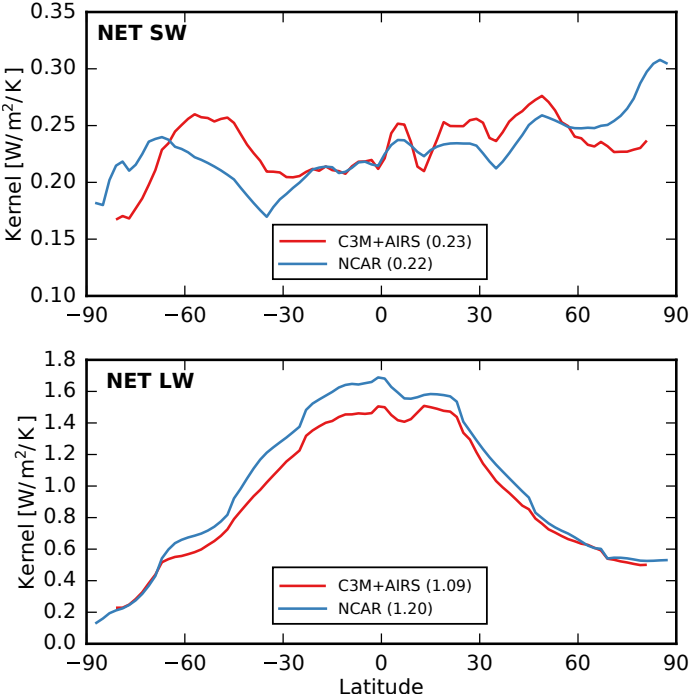


- Poorer C3M correlations mostly due to nadar-only sampling

Water vapor kernel

- Use CERES-PRP calculations to validate the NCAR CAM3 water vapor kernel (Shell et al. 2008)
- Follow the analysis of Soden et al. (2008) who documented differences in water vapor kernels among three different GCMs.

Water vapor kernel: vertically-integrated, annual-mean, zonal-mean



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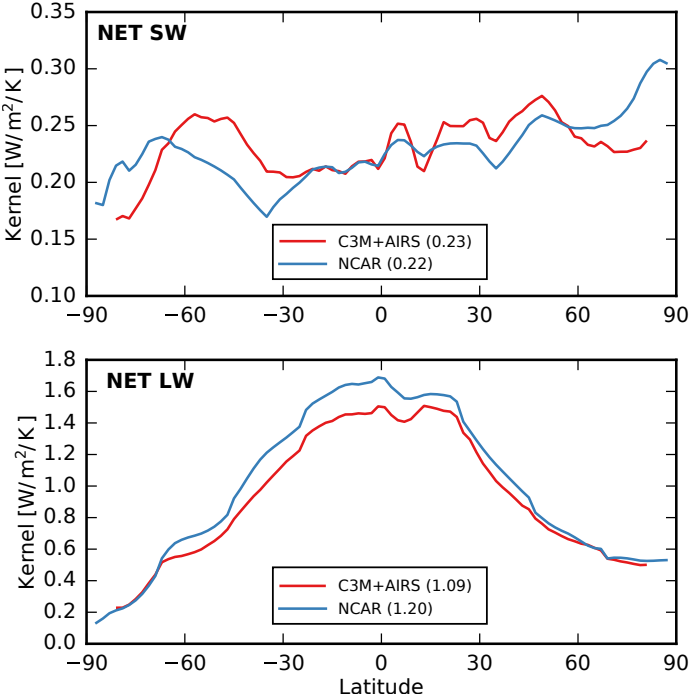

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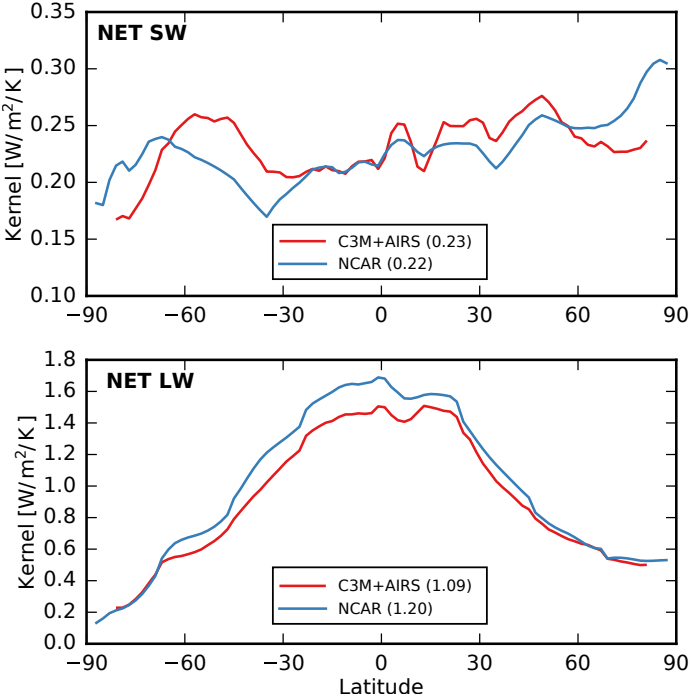

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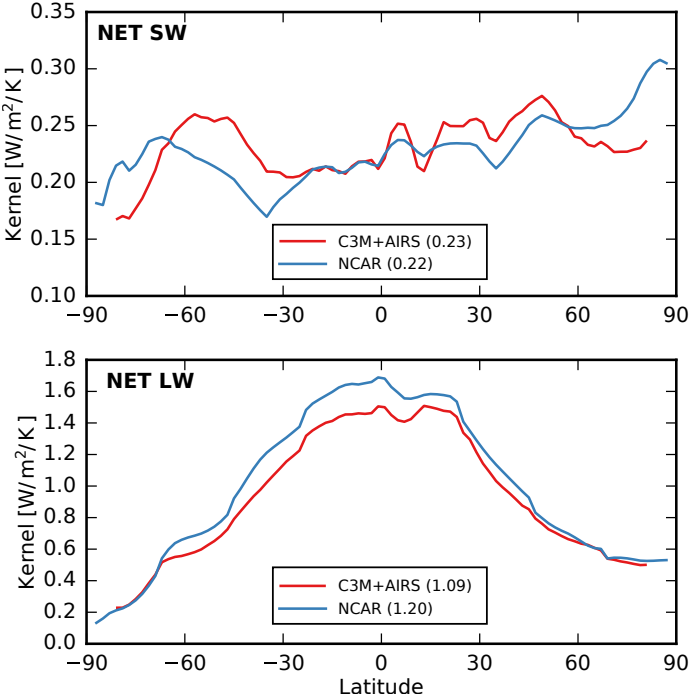

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- Errors are slightly larger than spread among GCMs in Soden et al (2008)
- Differences are still relatively modest, especially for global mean values

Conclusions

Using observations, decompose the total radiative flux monthly anomalies into the contributions from individual variables using PRP calculations

- Reasonably accurate to use monthly mean inputs
- Typically achieve an approximately linear decomposition of the total flux anomaly
- AIRS meteorological properties produce fluxes with smaller biases than GEOS
- Using C3M cloud properties produce fluxes with smaller biases than SYN
- SYN cloud properties are better at reproducing the variability
- Observation-based validation of the NCAR water vapor kernel: slightly larger errors than those implied by Soden et al. (2008)

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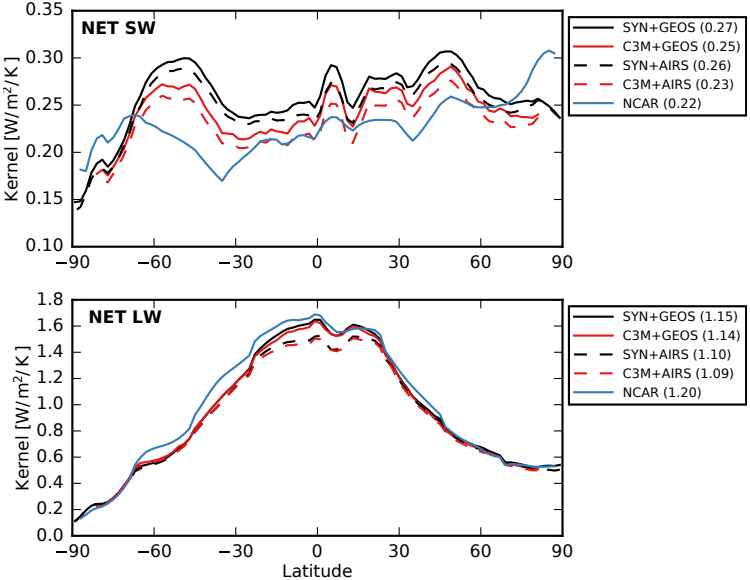

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- Kernels are within:
  - 11-39% for annual-mean zonal-mean values (not shown)
  - 13% for vertical-integrated annual-mean zonal-mean values
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- Errors are slightly larger than those inferred from different GCMs in Soden et al (2008)
- However, these differences are still relatively modest, especially for global mean values

- Compared to using GEOS (solid), AIRS (dashed) gives less sensitive kernels
- Compared to using SYN (black), C3M (red) gives **slightly** less sensitive kernels

Clouds

- Clouds need to be organized "radiatively" so that the radiative response to perturbations are approximately linear
  - Needed for valid monthly averages
  - Desirable to decompose cloud ∂F by type
  - Need this to make kernels

- Nominal SYN clouds already organized this way:
  4 non-overlapping single layer clouds (top > 700mb, 500-700mb, 300–500mb, <300mb)
  - Increase this to 12 cloud conditions by adding 3 optical depth categories (<1, 1–5, >5)

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  - Increase this to 12 cloud conditions by adding 3 optical depth categories (<1, 1–5, >5)
- C3M: clustering method groups and averages up to 40 monthly mean conditions (which vary both spatially and seasonally)

Error from monthly mean inputs

		TOA Ref. flux	TOA RMSE	TOA Bias			Surface Ref. flux	Surface RMSE	Surface Bias
Clear-sky	SW	**50.93**	**3.09**	**-1.36**	Clear-sky	SW	**243.63**	**1.32**	**-0.81**
	LW	**263.41**	**2.05**	**-1.01**		LW	**319.67**	**6.52**	**-5.05**
SYN CRE	SW	**50.03**	**2.25**	**0.31**	SYN CRE	SW	**-58.92**	**2.26**	**-0.60**
	LW	**-24.59**	**1.44**	**0.16**		LW	**29.05**	**2.28**	**1.65**
C3M CRE	SW	**47.57**	**0.88**	**-0.72**	C3M CRE	SW	**-56.55**	**1.06**	**0.89**
	LW	**-28.05**	**1.24**	**0.95**		LW	**31.51**	**0.57**	**-0.07**

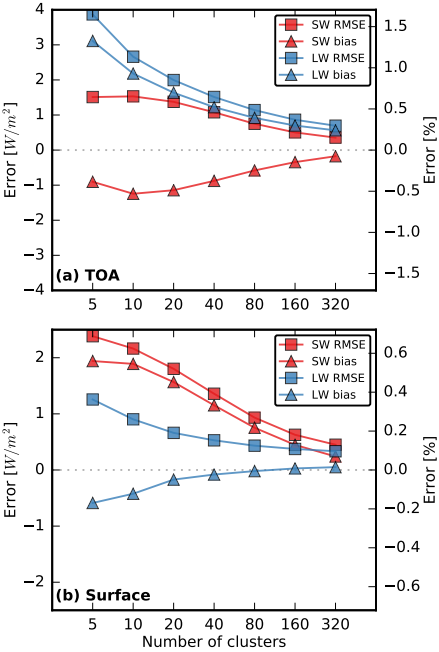
Figure: Global mean root mean square errors (RMSE) and bias errors $[W/m^2]$

- Reference ("Ref." fluxes: fluxes computed at their full temporal resolution and then averaged into monthly means
- Errors due to averaging of cloud properties isolated using the cloud radiative effect (CRE)

- Errors in TOA fluxes $< 6\%$.
- Errors in the surface fluxes $< 2\%$ for meteorological properties / C3M clouds; $< 8\%$ for SYN clouds
- Reasonable trade-off for the large decrease in computational time

C3M clustering

- Extend the methodology already used by C3M (Kato et al. 2010): average together similar cloud profiles
- Apply K-means clustering to the cloud base, top and (log) optical depth
- Separate clusters for each grid box, month of the year, and number of layers (1-6)
- After cluster centers are determined: each cloud profile matched to the nearest center and averaged into monthly means



Comparison to EBAF fluxes

	(a) All-sky				(b) Clear-sky				(c) CRE			
	SW		LW		SW		LW		SW		LW	
EBAF	**99.03**		**240.99**		**52.79**		**269.15**		**46.24**		**-28.16**	
SYN+GEOS	**4.04**	**0.87**	**3.97**	**-3.02**	**4.50**	**-3.22**	**7.51**	**-6.75**	**6.28**	**4.09**	**4.75**	**3.73**
C3M+GEOS	**5.26**	**-2.68**	**6.15**	**-5.43**	**4.54**	**-3.28**	**7.25**	**-6.50**	**4.58**	**0.61**	**2.54**	**1.07**
SYN+AIRS	**4.06**	**1.09**	**2.93**	**-0.72**	**4.47**	**-3.17**	**4.48**	**-3.68**	**6.41**	**4.27**	**3.81**	**2.97**
C3M+AIRS	**5.51**	**-3.32**	**3.95**	**-3.15**	**4.79**	**-3.69**	**4.27**	**-3.42**	**4.49**	**0.37**	**2.41**	**0.27**
	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias

Figure: The global mean root mean square errors (RMSE) and bias errors $[W/m^2]$ relative to the CERES EBAF-TOA fluxes

- The largest biases are in the clear-sky
  - The use of AIRS instead of GEOS reduces clear-sky longwave flux errors by about a factor of 2.
- Better agreement in the all-sky fluxes for the SYN cloud properties rather than C3M
  - Cloud radiative effect (CRE) comparisons: C3M does indeed provide a better agreement

Linearity test

For these calculations to be useful, the sum of the difference between the sum of individual perturbations ($\sum_i \partial F_{\Delta x_i}$) and the total anomaly ($\Delta F = F - \overline{F}$) must be small

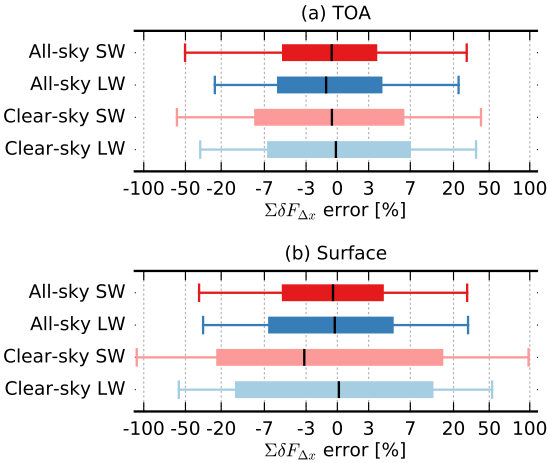


Figure: SYN+GEOS

- Median errors are mostly near-zero and IQR within about $\pm 10\%$
- Clear-sky shortwave errors are larger: median of -4% and IQR about $\pm 20\%$
- Overall, an approximately linear decomposition of the total flux anomalies

Linearity test

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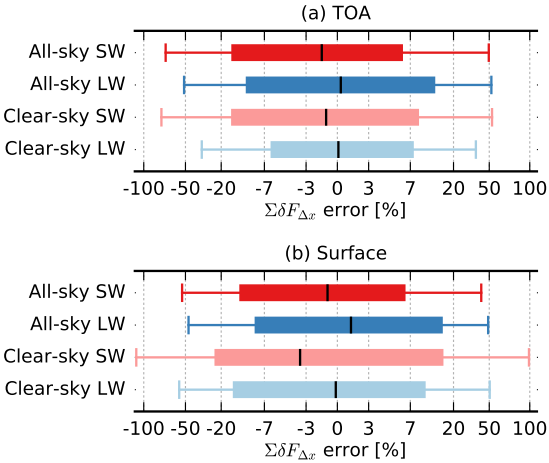


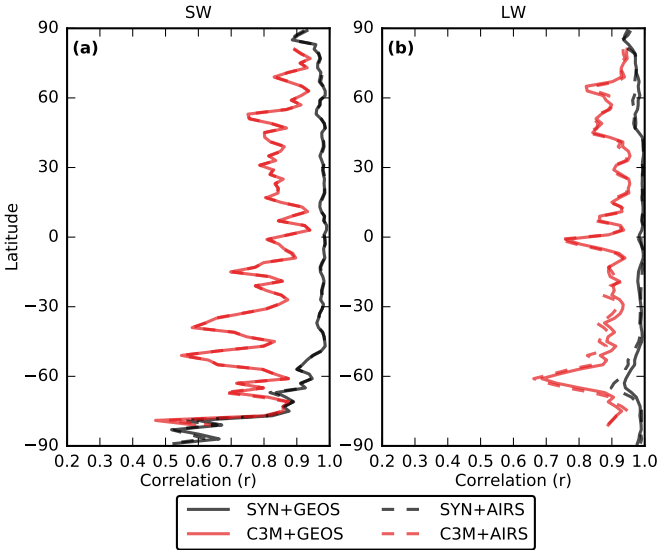
Figure: C3M+AIRS

- Median errors are mostly near-zero and IQR within about $\pm 10\%$
- Clear-sky shortwave errors are larger: median of -4% and IQR about $\pm 20\%$
- Overall, an approximately linear decomposition of the total flux anomalies

- Larger errors for C3M+GEOS all-sky flux, probably due to the increased complexity in organizing the C3M cloud conditions

Comparison to EBAF fluxes

- Correlation coefficients between the EBAF-TOA zonal mean times series and our computed fluxes

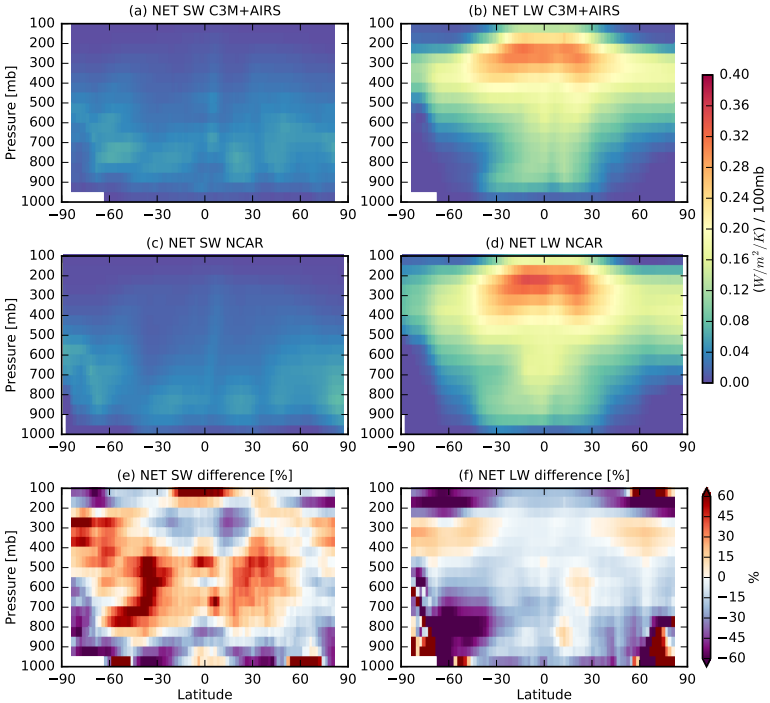


- Choice of GEOS (solid) or AIRS (dashed) has little effect on agreement
- SYN cloud properties (black) produce fluxes that correlate well with EBAF and outperform C3M (red)

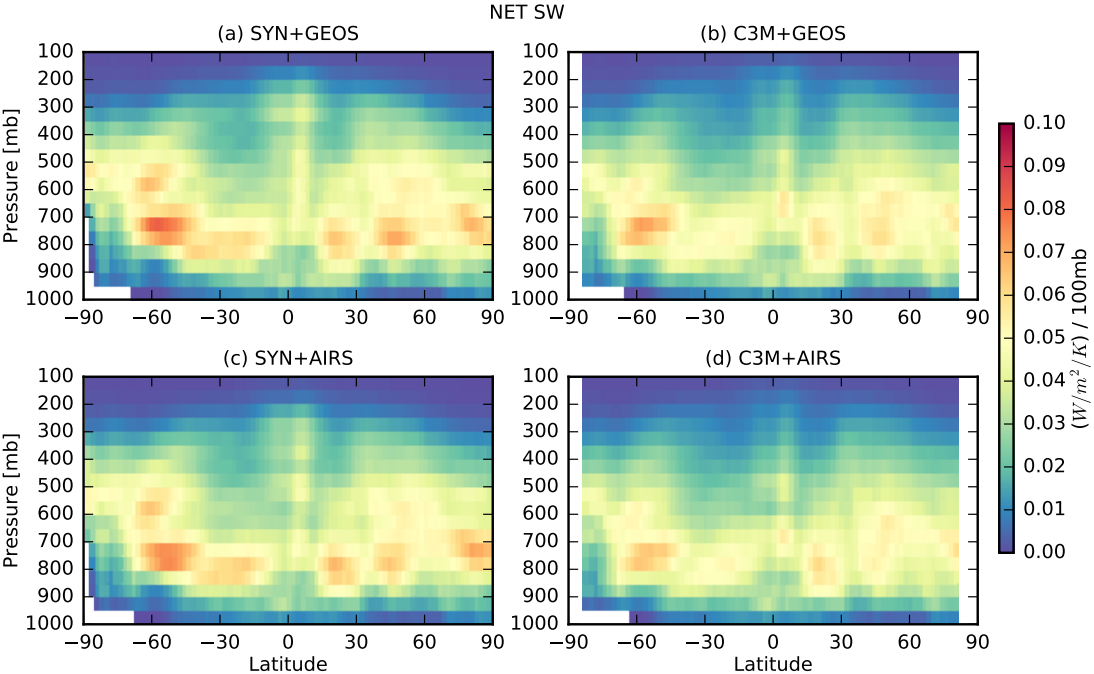
Diurnal cycle

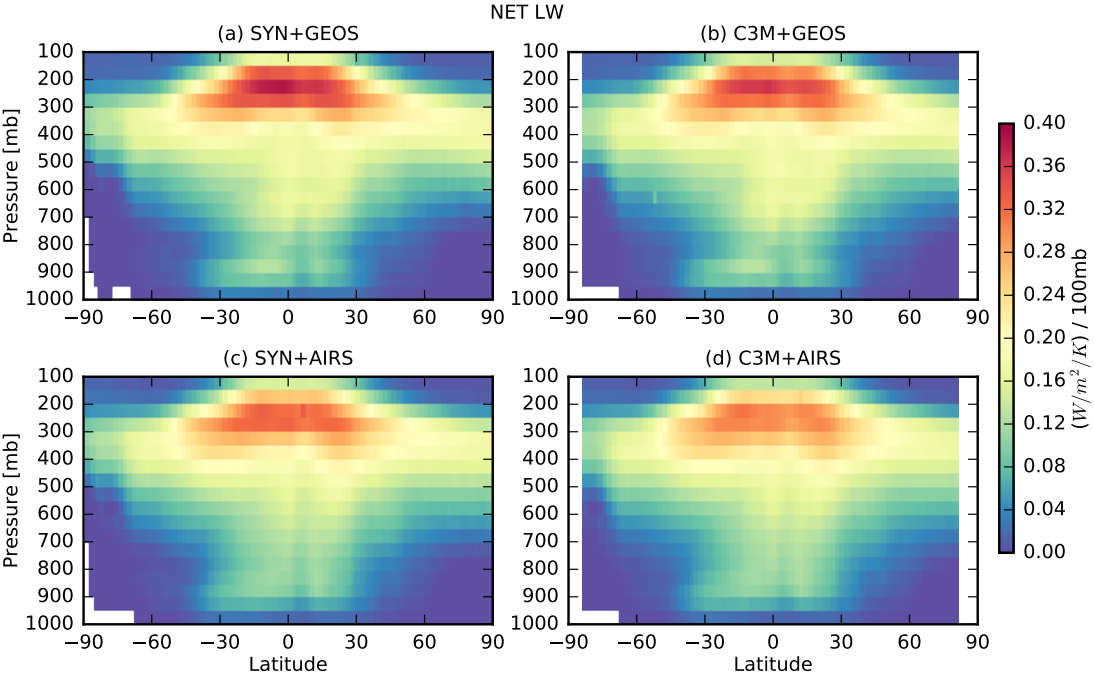
- Full representation of the diurnal cycle in the SYN/GEOS properties, but not in the C3M/AIRS properties
- Make calculations using a monthly mean diurnal cycle: some work to mix-and-match the different diurnal resolutions

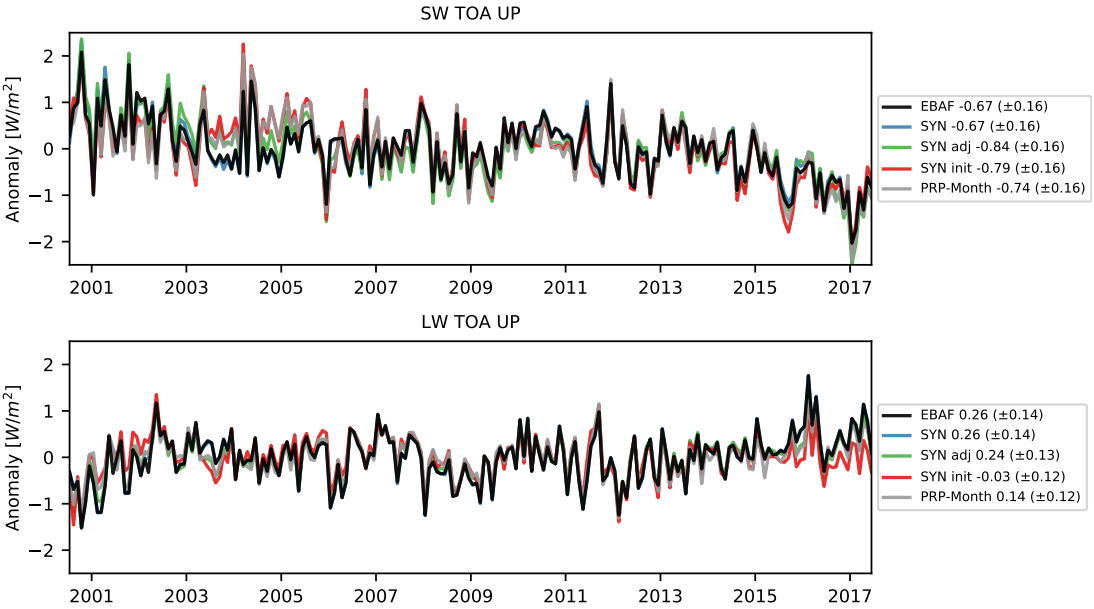
Water vapor kernel

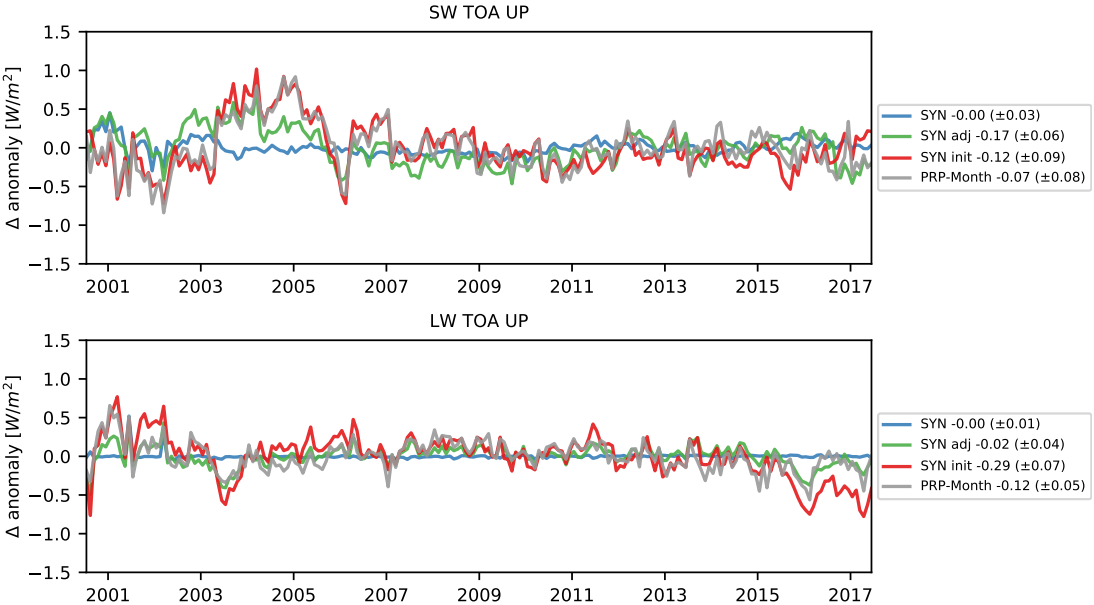


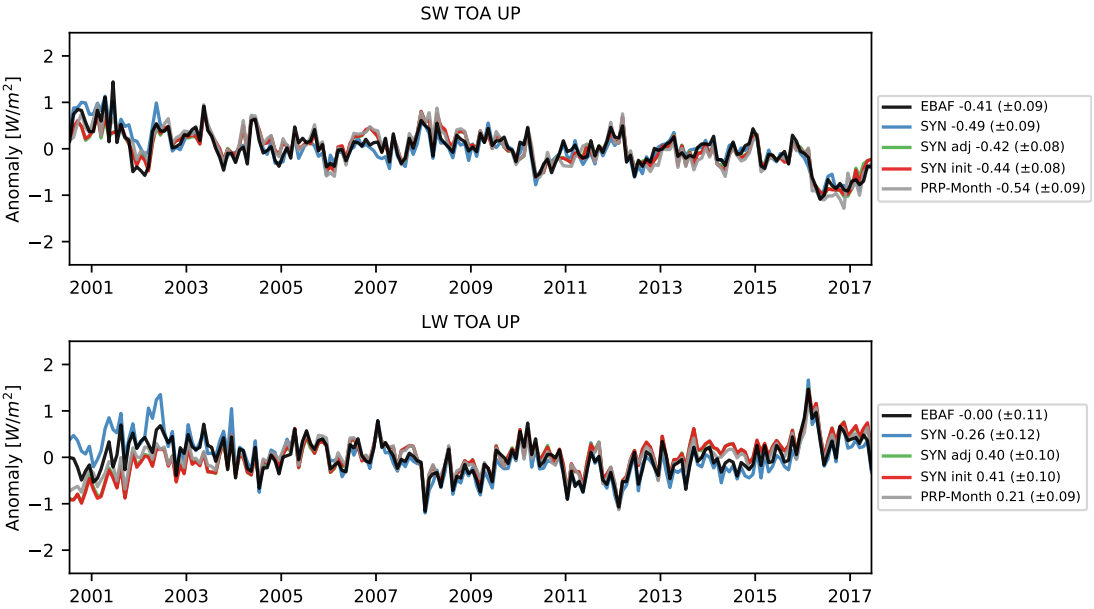
- C3M+AIRS kernel (best accuracy) vs. NCAR kernel
- Longwave response 4-5× > shortwave response
- The longwave is most sensitive to water vapor perturbations in the tropical upper troposphere
- Can be sizable regional differences...

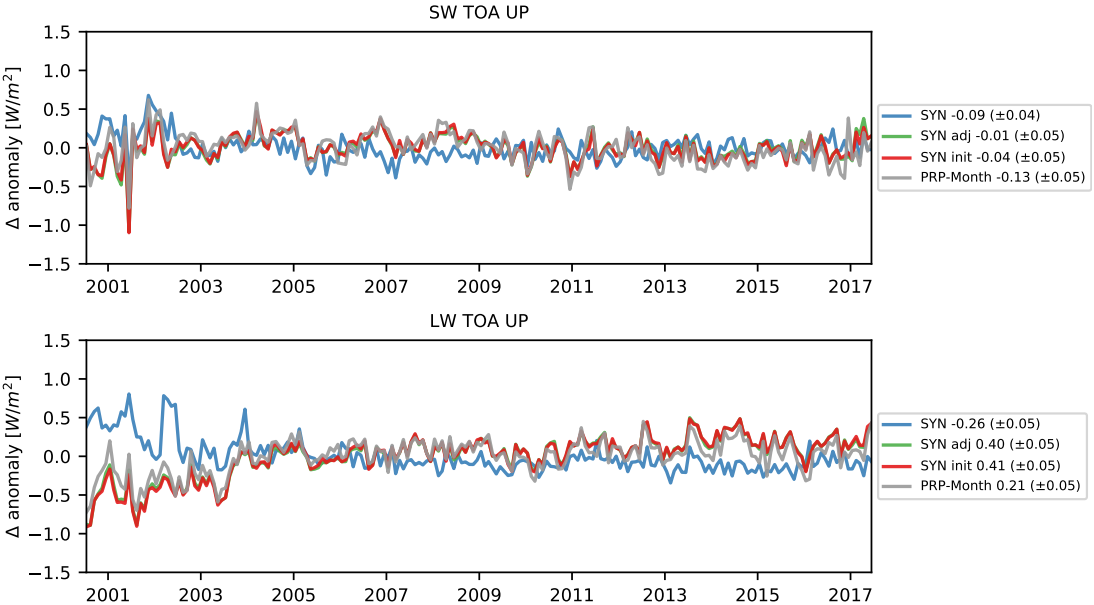

NET SW
(a) SYN+GEOS
(b) C3M+GEOS
(c) SYN+AIRS
(d) C3M+AIRS
Pressure [mb]
Latitude
100
200
300
400
500
600
700
800
900
1000
−90
−60
−30
0
30
60
90
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
$(W/m^2/K)$ / 100mb


NET LW
(a) SYN+GEOS
(b) C3M+GEOS
(c) SYN+AIRS
(d) C3M+AIRS
Pressure [mb]
Latitude
100
200
300
400
500
600
700
800
900
1000
−90
−60
−30
0
30
60
90
0.40
0.36
0.32
0.28
0.24
0.20
0.16
0.12
0.08
0.04
0.00
$(W/m^2/K)$ / 100mb


SW TOA UP
Anomaly [W/m²]
2
1
0
−1
−2
2001
2003
2005
2007
2009
2011
2013
2015
2017
EBAF -0.67 (±0.16)
SYN -0.67 (±0.16)
SYN adj -0.84 (±0.16)
SYN init -0.79 (±0.16)
PRP-Month -0.74 (±0.16)
LW TOA UP
Anomaly [W/m²]
2
1
0
−1
−2
2001
2003
2005
2007
2009
2011
2013
2015
2017
EBAF 0.26 (±0.14)
SYN 0.26 (±0.14)
SYN adj 0.24 (±0.13)
SYN init -0.03 (±0.12)
PRP-Month 0.14 (±0.12)


SW TOA UP
Δ anomaly [W/m²]
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
2001
2003
2005
2007
2009
2011
2013
2015
2017
SYN -0.00 (±0.03)
SYN adj -0.17 (±0.06)
SYN init -0.12 (±0.09)
PRP-Month -0.07 (±0.08)
LW TOA UP
Δ anomaly [W/m²]
SYN -0.00 (±0.01)
SYN adj -0.02 (±0.04)
SYN init -0.29 (±0.07)
PRP-Month -0.12 (±0.05)


SW TOA UP
Anomaly [W/m²]
2
1
0
−1
−2
2001
2003
2005
2007
2009
2011
2013
2015
2017
EBAF -0.41 (±0.09)
SYN -0.49 (±0.09)
SYN adj -0.42 (±0.08)
SYN init -0.44 (±0.08)
PRP-Month -0.54 (±0.09)
LW TOA UP
Anomaly [W/m²]
EBAF -0.00 (±0.11)
SYN -0.26 (±0.12)
SYN adj 0.40 (±0.10)
SYN init 0.41 (±0.10)
PRP-Month 0.21 (±0.09)


SW TOA UP
Δ anomaly [W/m²]
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
2001
2003
2005
2007
2009
2011
2013
2015
2017
SYN -0.09 (±0.04)
SYN adj -0.01 (±0.05)
SYN init -0.04 (±0.05)
PRP-Month -0.13 (±0.05)
LW TOA UP
Δ anomaly [W/m²]
SYN -0.26 (±0.05)
SYN adj 0.40 (±0.05)
SYN init 0.41 (±0.05)
PRP-Month 0.21 (±0.05)

PRP calculations

The effect on the flux (∂F) due to some perturbation Δx of variable x is

$$\partial F^f_{\Delta x} = F(x + \Delta x, y_1, ..., y_N) - F(x, y_1, ..., y_N) + O^f(\Delta x).$$

- F: flux at some level of the atmosphere
- x: monthly mean value of the variable of interest
- Δx: is the deseasonalized anomaly in x relative to the climatological month mean $\overline{x}$
  - $\Delta x = x - \overline{x}$
- Flux perturbation is relative to the monthly mean base state $(x, y_1, ..., y_N)$.
- Superscript f: forward finite difference

Alternatively, the effect of perturbation Δx can be computed using the backwards finite difference:

$$\partial F^b_{\Delta x} = F(x, y_1, ..., y_N) - F(x - \Delta x, y_1, ..., y_N) + O^b(\Delta x)$$

PRP calculations / radiative kernels

$$\partial F^f_{\Delta x} = F(x + \Delta x, y_1, ..., y_N) - F(x, y_1, ..., y_N) + O^f(\Delta x). \quad (1)$$

$$\partial F^b_{\Delta x} = F(x, y_1, ..., y_N) - F(x - \Delta x, y_1, ..., y_N) + O^b(\Delta x) \quad (2)$$

- Problem: Eqs. (1) and (2) have truncation errors that are the same order as the perturbation itself (Colman and McAvaney 1997, Soden et al. 2008)
- Instead, use the centered finite difference:

$$\partial F_{\Delta x} = \frac{F(x + \Delta x, y_1, ..., y_N) - F(x - \Delta x, y_1, ..., y_N)}{2} + O(\Delta x^2) \quad (3)$$

- Or minimize the truncation error by fixing Δx to a small perturbation δx in Eq. (1)

$$K^f_{\delta x} = \frac{\partial F^f_{\delta x}}{\delta x} \quad (4)$$

- This radiative kernel K^f that can then be multiplied by a larger Δx to obtain the corresponding radiative effect.

PRP calculations

$$\partial F^f_{\Delta x,M} = F(x+\Delta x, y_1, ..., y_N) - F(x, y_1, ..., y_N) + O^f_M(\Delta x) \quad (1)$$

$$\partial F^b_{\Delta x,M} = F(x, y_1, ..., y_N) - F(x-\Delta x, y_1, ..., y_N) + O^b_M(\Delta x) \quad (2)$$

- Possible for $x+\delta x$ and/or $x-\delta x$ to be non physical
- In that case, compute the perturbation relative to the climatological monthly means

$$\partial F^f_{\Delta x,C} = F(\overline{x}+\Delta x, \overline{y_1}, ..., \overline{y_N}) - F(\overline{x}, \overline{y_1}, ..., \overline{y_N}) + O^f_C(\Delta x), \quad (3)$$

$$\partial F^b_{\Delta x,C} = F(\overline{x}, \overline{y_1}, ..., \overline{y_N}) - F(\overline{x}-\Delta x, \overline{y_1}, ..., \overline{y_N}) + O^b_C(\Delta x) \quad (4)$$

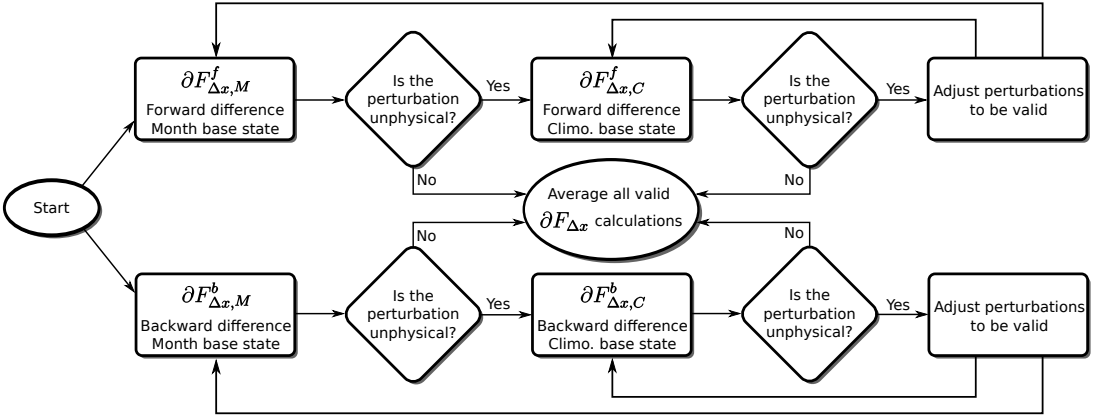


Figure: Flow diagram showing how the forward/backward finite differences using the monthly and climatological ("Climo.") mean base states are combined in CERES-PRP. The partial derivatives are defined in section **??**.

MATCH aerosols

- Model of Atmospheric Transport and Chemistry (MATCH) MODIS aerosol assimilation product (Collins et al. 2001)
- Aerosol optical depths (at 0.55 μm) and vertical distributions for seven different types of aerosols:
  1. $< 0.5\mu m$ dust[1]
  2. $> 0.5\mu m$ dust[1]
  3. Sulfate[2]
  4. Maritime[3]
  5. Soot[2]
  6. Hydrophilic organic carbon[2]
  7. Hydrophobic organic carbon[2]

[1]modified Tegen and Lacis 1996; [2]OPAC, Hess et al. 1998; [3]d'Almedia et al. 1991